JNOTES

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矢量

叉乘:

$\vec{A}\times\vec{B} \Rightarrow$ 三阶行列式: $\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

标量三重积(混合积)

$A\cdot(B\times C) = B\cdot(C\times A) = C\cdot(A\times B)$ 循环位移

矢量三重积的性质:

$A\times(B\times C) = B(A\cdot C) - C(A\cdot B)$

三种常见的正交坐标系

1.直角坐标系

位置矢量: $\vec{r} = x\vec{e}_x + y\vec{e}_y + z\vec{e}_z$

微分元矢量: $d\vec{r} = \vec{e}_x\cdot dx + \vec{e}_y dy + \vec{e}_z dz$

面积元: $dS_x = dydz$　$dS_y = dxdz$　$dS_z = dxdy$

体积元: $dV = dxdydz$.

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2. 圆柱坐标系

$0 \le \rho < \infty,\quad 0 \le \phi \le 2\pi,\quad -\infty < z < \infty$

$$\begin{cases} x = \rho\cos\phi \\ y = \rho\sin\phi \\ z = z \end{cases} \qquad \begin{aligned} \rho &= \sqrt{x^2+y^2} \\ \phi &= \arctan\left(\frac{y}{x}\right) \\ z &= z \end{aligned}$$

位置矢量: $\vec{r} = \rho\,\vec{e}_\rho + z\cdot\vec{e}_z$

坐标: (ρ, ϕ, z)

分解: $A = \vec{e}_\rho A_\rho + \vec{e}_\phi A_\phi + \vec{e}_z A_z$ → 分别在 $\vec{e}_\rho, \vec{e}_\phi, \vec{e}_z$ 方向上的投影

记忆: $\vec{e}_\phi = \vec{e}_x\cos\left(\frac{\pi}{2}-\phi\right) + \vec{e}_y\sin\left(\frac{\pi}{2}-\phi\right)$

$\vec{e}_\rho = \vec{e}_x\cos\phi + \vec{e}_y\sin\phi \qquad \vec{e}_\phi = \underline{-\vec{e}_x\sin\phi + \vec{e}_y\cos\phi}$

$\vec{e}_x = \vec{e}_\rho\cos\phi - e_\phi\sin\phi \qquad \vec{e}_y = \vec{e}_\rho\sin\phi + \vec{e}_\phi\cos\phi$

写成矩阵形式: 换能写成矩阵形式

$$\begin{bmatrix} e_\rho \\ e_\phi \\ e_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}$$

cosϕ 不变
sinϕ 变号

$$\begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} e_\rho \\ e_\phi \\ e_z \end{bmatrix}$$

$\vec{e}_\rho$ 和 $\vec{e}_\phi$ 是随 ϕ 变化 并非常矢量

$$\begin{cases} \dfrac{\partial \vec{e}_\rho}{\partial\phi} = -e_x\sin\phi + e_y\cos\phi = \vec{e}_\phi \\[2ex] \dfrac{\partial \vec{e}_\phi}{\partial\phi} = -e_x\cos\phi - e_y\sin\phi = -\vec{e}_\rho \end{cases}$$

不在同一点上直接代入矩阵运算

对于位于同一个点的两个矢量才可以用对应分量进行加法/乘法运算.

计算法则类似于直角坐标系

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位置矢量 $\vec{r} = \rho \cdot \vec{e}_\rho + z \cdot \vec{e}_z$

微元矢量 $d\vec{r} = d(\rho \cdot \vec{e}_\rho) + d(z \cdot \vec{e}_z)$

$= \vec{e}_\rho d\rho + \rho d\vec{e}_\rho + \vec{e}_z dz$

$= \vec{e}_\rho d\rho + \vec{e}_\phi \cdot \rho d\phi + \vec{e}_z dz$

长度元分别是 $dl_\rho = d\rho$ $dl_\phi = \rho d\phi$ $dl_z = dz$

拉梅系数 $h_\rho = \frac{dl_\rho}{d\rho} = 1$ $h_\phi = \frac{dl_\phi}{d\phi} = \rho$ $h_z = \frac{dl_z}{dz} = 1$

三个面积元: $dS_\rho = \rho d\phi dz$ $dS_\phi = d\rho dz$ $dS_z = \rho d\rho d\phi$

体积元: $dV = \rho d\rho d\phi dz$

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3. 球坐标系

俯仰角　　方位角

$0 \le r < \infty \quad 0 \le \theta \le \pi \quad 0 \le \phi \le 2\pi$

$r = \sqrt{x^2+y^2+z^2} \quad \theta = \arccos\left(z/\sqrt{x^2+y^2+z^2}\right) \quad \phi = \arctan(y/x)$

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases}$$

$\vec{e_r} = \vec{e_x}\cdot\sin\theta\cos\phi + \vec{e_y}\sin\theta\sin\phi + \vec{e_z}\cos\theta$

$\vec{e_\theta} = \vec{e_x}\cdot\cos\theta\cos\phi + \vec{e_y}\cdot\cos\theta\sin\phi - \vec{e_z}\cdot\sin\theta$

$\vec{e_\phi} = \vec{e_x}\cdot(-\sin\phi) + \vec{e_y}\cdot\cos\phi + \vec{e_z}\cdot 0$

对θ求偏导

对φ求偏导 再除sinθ.

$$\begin{cases} \frac{\partial \vec{e_r}}{\partial\theta} = \vec{e_x}\cos\theta\cos\phi + \vec{e_y}\cos\theta\sin\phi - \vec{e_z}\sin\theta = \vec{e_\theta}. \\ \frac{\partial \vec{e_r}}{\partial\phi} = -\vec{e_x}\sin\theta\sin\phi + \vec{e_y}\sin\theta\cos\phi = \vec{e_\phi}\cdot\sin\theta. \end{cases}$$

$$\begin{cases} \frac{\partial \vec{e_\theta}}{\partial\theta} = -\vec{e_x}\sin\theta\cos\phi - \vec{e_y}\sin\theta\sin\phi - \vec{e_z}\cdot\cos\theta = -\vec{e_r}. \\ \frac{\partial \vec{e_\theta}}{\partial\phi} = -\vec{e_x}\cos\theta\sin\phi + \vec{e_y}\cos\theta\cos\phi = \vec{e_\phi}\cdot\cos\theta \end{cases}$$

$$\begin{cases} \frac{\partial \vec{e_\phi}}{\partial\theta} = 0 \\ \frac{\partial \vec{e_\phi}}{\partial\phi} = -\vec{e_r}\sin\theta - \vec{e_\theta}\cos\theta \end{cases}$$

位置矢量: $\vec{r} = \vec{e_r}\, r$

拉梅系数 $\begin{cases} h_r = 1 \\ h_\theta = r \\ h_\phi = r\sin\theta \end{cases}$

线元: $d\vec{r} = r\,d\vec{e_r} + \vec{e_r}\,dr$

$= r(\vec{e_\theta}\,d\theta + \vec{e_\phi}\sin\theta\,d\phi) + \vec{e_r}\,dr$

$= \vec{e_r}\,dr + \underline{r}\,\vec{e_\theta}\,d\theta + \underline{r\sin\theta}\,\vec{e_\phi}\,d\phi$

看这儿!

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面元： $d\vec{S}_r = \vec{e}_r \cdot dS_r = r^2 \sin\theta d\theta d\phi$

$d\vec{S}_\theta = r\sin\theta dr d\phi$

$d\vec{S}_\phi = r dr d\theta$

体积元： $dV = dr \cdot r d\theta \cdot r\sin\theta d\phi$

$= r^2 \sin\theta dr d\theta d\phi$

标量场的方向导数与梯度

1. 等值面. 等值面方程 $u(r)=C$

2. 方向导数:

$$\left.\frac{\partial u}{\partial l}\right|_{M_0}=\lim_{\Delta l\to 0}\frac{u(M)-u(M_0)}{\Delta l}$$

即: $\frac{\partial u}{\partial l}=\frac{\partial u}{\partial x}\cos\alpha+\frac{\partial u}{\partial y}\cos\beta+\frac{\partial u}{\partial z}\cos\gamma$

$\cos\alpha.\ \cos\beta.\ \cos\gamma$ 是 l 的方向余弦

$\vec{e}_l\cdot\left(\frac{\partial u}{\partial x},\frac{\partial u}{\partial y},\frac{\partial u}{\partial z}\right)=\frac{\partial u}{\partial l}$ $\Rightarrow\left(\frac{\partial}{\partial x}\cdot\vec{e}_x+\frac{\partial}{\partial y}\vec{e}_y+\frac{\partial}{\partial z}\vec{e}_z\right)\cdot u$

$\vec{e}_l\cdot \mathrm{grad}\,u=\frac{\partial u}{\partial l}$

梯度

$\boxed{矢}$量微分算子

哈密顿算符: ∇ ("del" 或 "Nabla")

圆柱坐标系: $\nabla u=\vec{e}_\rho\frac{\partial u}{\partial\rho}+\vec{e}_\phi\frac{1}{\rho}\cdot\frac{\partial u}{\partial\phi}+\vec{e}_z\frac{\partial u}{\partial z}$

球坐标系: $\nabla u=\vec{e}_r\cdot\frac{\partial u}{\partial r}+\vec{e}_\theta\cdot\frac{1}{r}\frac{\partial u}{\partial\theta}+\vec{e}_\phi\cdot\frac{1}{r\sin\theta}\frac{\partial u}{\partial\phi}$

拉梅系数倒数

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梯度的基本公式(类似于求导)

$$\nabla(Cu) = C\nabla u$$

$$\nabla(u \pm v) = \nabla u \pm \nabla v$$

$$\nabla(uv) = u\nabla v + v\nabla u$$

$$\nabla\left(\frac{u}{v}\right) = \frac{1}{v^2}\left(v\nabla u - u\nabla v\right)$$

$$\nabla f(u) = f'(u)\nabla u$$

矢量场的通量与散度

1. 通量:(标量)

$$\Phi = \int d\Phi = \int_S \vec{F}\cdot d\vec{S} = \int_S \vec{F}\cdot \vec{e}_n dS$$

$\Phi > 0$ 穿出 > 穿入，表明闭合面中有正源(发出)

$\Phi < 0$ 穿出 < 穿入，表明 …… —— 负源(汇集)

$\Phi = 0$ 穿出 = 穿入　无源　正负源代数和为0

2. 散度:(通量的密度)

$$\mathrm{div}\vec{F} = \lim_{\Delta V \to 0} \frac{\oint \vec{F}\cdot d\vec{S}}{\Delta V}$$

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$$\mathrm{div}\,F = \left(e_x\frac{\partial}{\partial x} + e_y\frac{\partial}{\partial y} + e_z\frac{\partial}{\partial z}\right)\cdot\left(e_xF_x + e_yF_y + e_zF_z\right)$$

$$= \nabla F$$

$\nabla\cdot\vec{F} = \rho \neq 0$ 有源场

$\nabla\cdot\vec{F} = 0$ 无源场

散度定理 (矢量场的高斯定理)

$$\oint_S \vec{F}\cdot d\vec{s} = \int_V \nabla\cdot\vec{F}\,dv$$

矢量场的环流与旋度

1. 环流：在闭合线上积分

$$\Gamma = \oint_C \vec{F}\cdot d\vec{l}$$

2. 环流面密度

$$\mathrm{rot}_n\vec{F} = \lim_{\Delta S\to 0}\frac{\oint_C \vec{F}\cdot d\vec{l}}{\Delta S}$$

$$\mathrm{rot}_n\vec{F} = \vec{e}_n\cdot \mathrm{rot}\,\vec{F}$$

$$\mathrm{rot}\,\vec{F} = \vec{e}_n\cdot\mathrm{rot}_n\vec{F} = e_x\mathrm{rot}_x\vec{F} + e_y\mathrm{rot}_y\vec{F} + e_z\mathrm{rot}_z\vec{F}$$

$\mathrm{rot}\,\vec{F} = \nabla\times\vec{F}$ 写成行列式

$$\nabla\times\vec{F} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

同理可得圆柱和球坐标系的表达式

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$rot_n\vec{F} = \vec{e_n}\cdot\nabla\times\vec{F}$

$\nabla\times\vec{F} = \vec{J} \neq 0$ 有旋场 $\nabla\times\vec{F} = \vec{J} = 0$ 无旋场

结论: $\nabla\cdot(\nabla\times\vec{F}) \equiv 0$ → 矢量场的旋度的散度恒为0

$\nabla\times(\nabla u) \equiv 0$ → 标量场梯度的旋度恒为0

斯托克斯定理(旋度定理)

$\oint_C \vec{F}\cdot d\vec{l} = \int_S \nabla\times\vec{F}\cdot d\vec{S}$

无旋场的标量位

$\nabla\times\vec{F} \equiv 0$ → 无旋场 $\oint_C \vec{F}\cdot d\vec{l} = 0$

又之前性质可知 $\nabla\times\nabla u \equiv 0$

∴ 可得出:

$\vec{F} = -\nabla u$ u 标量位函数

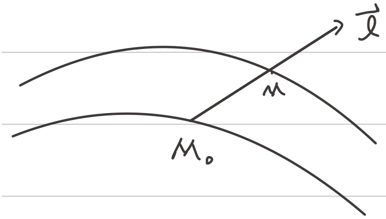
如何求位函数?

$\vec{F} \rightarrow u(M) = \int_M^{M_0} \vec{F}\cdot d\vec{l} + C$

由标量场的梯度求标量场

$\nabla u \rightarrow u(M) = -\int_M^{M_0} \nabla u\cdot d\vec{l} + C$

方向导数 $\frac{\partial u}{\partial l}$: 场量在 $\vec{l}$ 方向上的空间变化率



$\Delta u = u(M) - u(M_0)$

$\Delta l = \overline{MM_0}$

$$\boxed{\left.\frac{\partial u}{\partial l}\right|_{M_0} = \lim_{M\to M_0}\frac{\Delta u}{\Delta l}}$$

$$\vec{l} = l_x\vec{e}_x + l_y\vec{e}_y + l_z\vec{e}_z$$

$$\vec{l}_0 = \frac{\vec{l}}{|\vec{l}|} = \frac{l_x}{|\vec{l}|}\vec{e}_x + \frac{l_y}{|\vec{l}|}\vec{e}_y + \frac{l_z}{|\vec{l}|}\vec{e}_z = \cos\alpha\,\vec{e}_x + \cos\beta\,\vec{e}_y + \cos\gamma\,\vec{e}_z$$

$$\boxed{\frac{\partial u}{\partial l} = \frac{\partial u}{\partial x}\cos\alpha + \frac{\partial u}{\partial y}\cos\beta + \frac{\partial u}{\partial z}\cos\gamma}$$

$$\boxed{\vec{G} = \frac{\partial u}{\partial x}\vec{e}_x + \frac{\partial u}{\partial y}\vec{e}_y + \frac{\partial u}{\partial z}\vec{e}_z}$$

$\frac{\partial u}{\partial l} = \vec{G}\cdot\vec{l}_0 = |G|\cos\theta$ 当 $\cos\theta = 1$

梯度 $\mathrm{grad}\,u \triangleq \vec{G}$

① $\mathrm{grad}\,u \Rightarrow \max\frac{\partial u}{\partial l}$

② $\frac{\partial u}{\partial l}$ 由 $\mathrm{grad}\,u$ 确定

哈密顿算子: $\boxed{\nabla = \frac{\partial}{\partial x}\vec{e}_x + \frac{\partial}{\partial y}\vec{e}_y + \frac{\partial}{\partial z}\vec{e}_z}$ $\mathrm{grad}\,u = \nabla u$

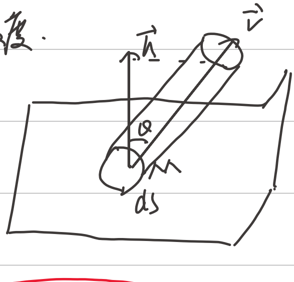
性质: ① $\nabla(Cu) = C\nabla u$ ② $\nabla(u+v) = \nabla u + \nabla v$

③ $\nabla(uv) = u\nabla v + v\nabla u$ (求导) ④ $\nabla\left(\frac{u}{v}\right) = \frac{1}{v^2}(u\nabla v - v\nabla u)$

⑤ $\nabla f(u) = f'(u)\cdot\nabla u$

通量与散度.

$\vec{v}$·流速



$\vec{dS} = dS\cdot\vec{n}$

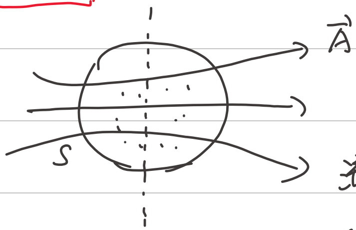
通过面元dS的流量:

$dQ = dS\cdot|v|\cos\theta = \vec{v}\cdot\vec{dS}$

求总流量 $Q = \int_S dQ = \int_S \vec{v}\cdot\vec{dS}$

通量. $\boxed{\Phi = \int_S \vec{A}\,\vec{dS}}$

若是闭合曲面



外侧规定为正

流出>流入 :有正源

流出<流入:有负源

散度: $div\vec{A}$:通量的体密度

$$\oint_S \vec{A}\cdot\vec{dS} = \oint_S A_x\,dy\,dz + A_y\,dx\,dz + A_z\,dx\,dy = \int_V \boxed{\left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right)} dV$$

$\vec{A} = A_x\vec{e}_x + A_y\vec{e}_y + A_z\vec{e}_z$ $\qquad div\vec{A} \triangleq \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

$\vec{dS} = dy\,dz\,\vec{e}_x + dx\,dz\,\vec{e}_y + dx\,dy\,\vec{e}_z$ $\qquad div\vec{A} = \nabla\cdot\vec{A}$

小结: ① $\nabla\cdot(C\vec{A}) = C(\nabla\cdot\vec{A})$ ② $\nabla\cdot(\vec{A}+\vec{B}) = \nabla\cdot\vec{A} + \nabla\cdot\vec{B}$

③ $\nabla\cdot(u\vec{A}) = \nabla u\cdot\vec{A} + u(\nabla\cdot\vec{A})$ (u:标量场)

$\boxed{\oint_S \vec{A}\cdot\vec{dS} = \int_V \nabla\cdot\vec{A}\,dV \quad \text{散度th.}}$

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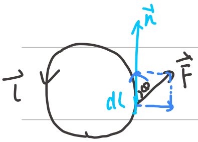
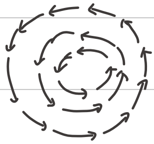
环量与旋度





旋涡场

旋涡源 / 环量源

$\vec{dl} = dl\cdot\vec{n}$ 做功如下:

$$dW = |F|\cdot\cos\theta\cdot dl = \vec{F}\cdot\vec{dl}$$

$$W = \oint_L dW = \oint_L \vec{F}\cdot\vec{dl} \qquad \boxed{\Gamma = \oint_L \vec{A}\cdot\vec{dl}}$$

2. 旋度 rot $\vec{A}$: 小量闭上. 环量面密度.

$$\oint_L \vec{A}\cdot\vec{dl} \qquad \vec{A} = A_x\vec{e}_x + A_y\vec{e}_y + A_z\vec{e}_z$$

$$\vec{dl} = dx\,\vec{e}_x + dy\,\vec{e}_y + dz\,\vec{e}_z$$

$$\oint_L \vec{A}\cdot\vec{dl} = \oint_L A_x dx + A_y dy + A_z dz$$ (stokes th.)

$$= \iint_S \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)dydz + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)dxdz + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)dxdy$$

$$\begin{cases} \vec{ds} = dydz\,\vec{e}_x + dxdz\,\vec{e}_y + dxdy\,\vec{e}_z \\ \boxed{\vec{R} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\vec{e}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\vec{e}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\vec{e}_z} = \text{rot}\,\vec{A} \end{cases}$$

$$\oint_L \vec{A}\cdot\vec{dl} = \iint_S \vec{R}\,\vec{ds}$$

环量面密度

算子形式: $\text{rot}\,\vec{A} = \nabla\times\vec{A} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$

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性质. ① $\nabla\times(\nabla u)=0$ 梯度无旋

② $\nabla\cdot(\nabla\times\vec{A})=0$ 旋度无散

③ $\nabla\times(u\vec{A})=u\nabla\times\vec{A}+\nabla u\times\vec{A}$

④ $\nabla\cdot(\vec{A}\times\vec{B})=B\cdot\nabla\times\vec{A}-A\cdot\nabla\times\vec{B}$

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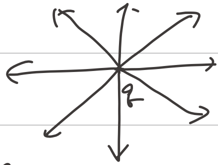
亥姆霍兹定理

1. 矢量场分类:

① 无旋场 $\vec{F_1}$: $\nabla\times\vec{F_1}=0$ 由 $\nabla\times(\nabla u)=0$ 得 $\vec{F_1}$ 可写作一个梯度场. 即:

$$\underline{\vec{F_1}=-\nabla u}$$ 势函数 (势场)

② 无散场 (无源场) $\vec{F_2}$:



$\nabla\cdot\vec{F_2}=0$

由 $\nabla\cdot(\nabla\times\vec{A})=0$ $\vec{F_2}$ 可写作一个旋度场. 即.

$\vec{F_2}=\nabla\times\vec{A}$

③ 调和场: 无旋 + 无散 F

$F=-\nabla u$　　$\nabla\cdot(\nabla u)=0$　　$\nabla^2 u=0$

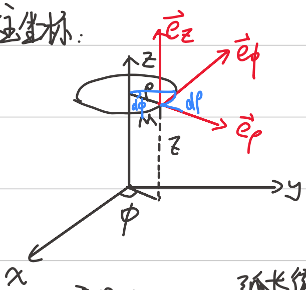
$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$ Laplace 算子

亥姆霍兹定理: 任意一个矢量场 $\vec{F}$ 可分解为一个无旋场和一个无散场的和

即: $\vec{F}=\vec{F_1}+\vec{F_2}$

柱坐标和球坐标:

1. 柱坐标:



坐标 (ρ, ϕ, z)

单位向量 $(\vec{e}_\rho, \vec{e}_\phi, \vec{e}_z)$

度量系数 (h_ρ, h_ϕ, h_z) （拉梅系数）

$(1, \rho, 1)$

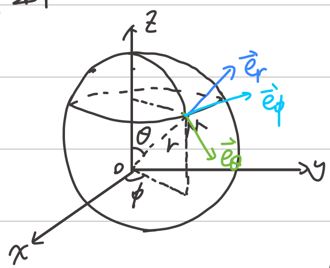
度量系数 $= \dfrac{\text{弧长微分}}{\text{坐标微分}}$ → 轨迹长度

$dl_\rho = d\rho \qquad h_\rho = \dfrac{dl_\rho}{d\rho} = 1$

$dl_\phi = \rho\, d\phi \qquad h_\phi = \dfrac{dl_\phi}{d\phi} = \rho$

$dl_z = dz \qquad h_z = \dfrac{dl_z}{dz} = 1$

2. 球坐标



坐标 (r, θ, ϕ)

单位向量 $(\vec{e}_r, \vec{e}_\theta, \vec{e}_\phi)$

度量系数 (h_r, h_θ, h_ϕ)

$(1, r, r\sin\theta)$

$dl_\theta = r \cdot d\theta \qquad h_\theta = \dfrac{dl_\theta}{d\theta} = r$

$dl_\phi = r\sin\theta\, d\phi \qquad h_\phi = \dfrac{dl_\phi}{d\phi} = r\sin\theta$

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$$\nabla^2\varphi=\frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}\left(\frac{h_2h_3}{h_1}\frac{\partial\varphi}{\partial u_1}\right)+\frac{\partial}{\partial u_2}\left(\frac{h_3h_1}{h_2}\frac{\partial\varphi}{\partial u_2}\right)+\frac{\partial}{\partial u_3}\left(\frac{h_1h_2}{h_3}\frac{\partial\varphi}{\partial u_3}\right)\right]$$

(Laplace 算子)

正交坐标系:

坐标 u_1, u_2, u_3
- 直角 (x, y, z)
- 柱 (ρ, ϕ, z)
- 球 (r, θ, ϕ)

单位向量 $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$
- 直角 $(\vec{e}_x, \vec{e}_y, \vec{e}_z)$
- 柱 $(\vec{e}_\rho, \vec{e}_\phi, \vec{e}_z)$
- 球 $(\vec{e}_r, \vec{e}_\theta, \vec{e}_\phi)$

度量系数 h_1, h_2, h_3
- 直角 $(1, 1, 1)$
- 柱 $(1, \rho, 1)$
- 球 $(1, r, r\sin\theta)$

线元矢量: $d\vec{l}=h_1du_1\vec{e}_1+h_2du_2\vec{e}_2+h_3du_3\vec{e}_3$

直角: $d\vec{l}=dx\,\vec{e}_x+dy\,\vec{e}_y+dz\,\vec{e}_z$

柱: $d\vec{l}=d\rho\,\vec{e}_\rho+\rho d\phi\,\vec{e}_\phi+dz\,\vec{e}_z$

球: $d\vec{l}=dr\,\vec{e}_r+rd\theta\,\vec{e}_\theta+r\sin\theta d\phi\,\vec{e}_\phi$

面元矢量: $d\vec{S}=h_2h_3du_2du_3\vec{e}_1+h_1h_3du_1du_3\vec{e}_2+h_2h_3du_2du_3\vec{e}_3$

$\nabla\varphi=\frac{1}{h_1}\frac{\partial\varphi}{\partial u_1}\vec{e}_1+\frac{1}{h_2}\frac{\partial\varphi}{\partial u_2}\vec{e}_2+\frac{1}{h_3}\frac{\partial\varphi}{\partial u_3}\vec{e}_3$ 梯度.

$\nabla\cdot\vec{A}=\frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}(h_2h_3A_1)+\frac{\partial}{\partial u_2}(h_1h_3A_2)+\frac{\partial}{\partial u_3}(h_1h_2A_3)\right]$ 散度

$$\nabla\times\vec{A}=\frac{1}{h_1h_2h_3}\begin{vmatrix}h_1\vec{e}_1 & h_2\vec{e}_2 & h_3\vec{e}_3\\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3}\\ h_1A_1 & h_2A_2 & h_3A_3\end{vmatrix}$$ 旋度

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散度定理, 旋度定理

直角: $\vec{A}(x,y,z) = A_x\vec{e}_x + A_y\vec{e}_y + A_z\vec{e}_z$　　体积分 ⇔ 三重积分

$dV = dx\,dy\,dz$　　通量运算 ⇔ 第二类面积分（对坐标）

$d\vec{S} = dy\,dz\,\vec{e}_x + dx\,dz\,\vec{e}_y + dx\,dy\,\vec{e}_z$　　环量运算 ⟺ 线积分（对坐标）

$d\vec{l} = dx\,\vec{e}_x + dy\,\vec{e}_y + dz\,\vec{e}_z$

高斯公式: $\iiint_\Omega \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\,dV = \oiint_\Sigma P\,dy\,dz + Q\,dz\,dx +$

令 $P = A_x(x,y,z)$　$Q = A_y(x,y,z)$　$R = A_z(x,y,z)$

则左边 $= \int_V \nabla\cdot\vec{A}\,dV$　散度体积分

右边 $= \oint_S \vec{A}\cdot d\vec{S}$　通量

故 $\int_V \nabla\cdot\vec{A}\,dV = \oint_S \vec{A}\cdot d\vec{S}$　散度定理

斯托克斯公式: $\iint_\Sigma (\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z})dy\,dz + (\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x})dx\,dz + (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})dx\,dy$

$= \oint_\Gamma P\,dx + Q\,dy + R\,dz$

左边 $= \int_S \nabla\times\vec{A}\cdot d\vec{S}$　　右边 $= \oint_L \vec{A}\cdot d\vec{l}$

$\int_S (\nabla\times\vec{A})\cdot d\vec{S} = \oint_L \vec{A}\cdot d\vec{l}$　旋度定理

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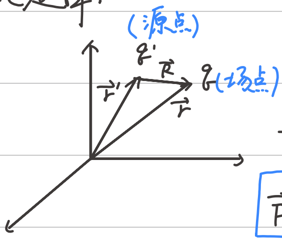
库仑定律.



$\vec{R} = \vec{r} - \vec{r}'$

$$\vec{F}(\vec{r}) = \frac{q'q}{4\pi\varepsilon_0} \frac{1}{R^2} \vec{n}_R$$

$$\boxed{\vec{F}(\vec{r}) = \frac{q'q}{4\pi\varepsilon_0} \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}}$$ 矢量式

2.电场强度 $\vec{E}(\vec{r}) = \frac{F(\vec{r})}{q}$

$$\vec{E}(\vec{r}) = \frac{q'}{4\pi\varepsilon_0} \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

3.分布电荷. 电荷密度 ρ

①体分布: ρ_V, $\vec{E}(\vec{r}) = \int_{V'} \frac{\rho_V(\vec{r}')}{4\pi\varepsilon_0} \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} dV'$

②面分布: ρ_S,

③线分布: ρ_L,

高斯定理:

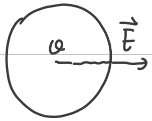
闭合曲面上的电通量只与其内部的电荷总量有关

$$\boxed{\oint \vec{E}\cdot d\vec{s} = \frac{Q}{\varepsilon_0}}$$

$$\oint \vec{E}\cdot d\vec{s} = \int_V \nabla\cdot\vec{E}\,dV$$ (散度th.)

$$\frac{Q}{\varepsilon_0} = \int_V \frac{\rho(r)}{\varepsilon_0}\,dV \Rightarrow \boxed{\underbrace{\boxed{\nabla\cdot\vec{E}}}_{散度} = \frac{\rho(r)}{\varepsilon_0}}$$ (微分形式)

① 球面+点电荷

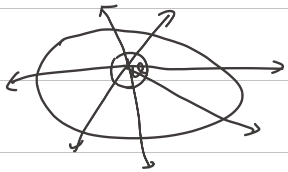


$$\oint \vec{E}\cdot d\vec{s} = \oint_s |\vec{E}|\,ds = \oint_s \frac{Q}{4\pi\varepsilon_0}\frac{1}{R^2}\,ds$$

$$= \frac{Q}{4\pi\varepsilon_0 R^2}\oint_s ds$$

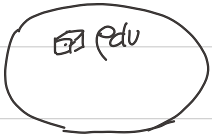
$$= \frac{Q}{4\pi\varepsilon_0 R^2}\cdot 4\pi R^2 = \frac{Q}{\varepsilon_0}$$

② 闭合曲面内+点电荷

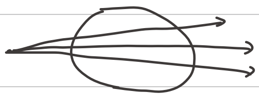


③ 闭合曲面内+分布电荷:

$$\int_V \frac{\rho\,dV}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$



④ 闭合曲面外+分布电荷:



通量为0。

常用公式: $\dfrac{\vec r-\vec r'}{|\vec r-\vec r'|^3} = -\nabla \dfrac{1}{|\vec r-\vec r'|}$

电压:

$$\vec E(\vec r) = \int_{V'} \frac{\rho(\vec r')}{4\pi\varepsilon_0}\frac{\vec r-\vec r'}{|\vec r-\vec r'|^3}\,dV'$$

$$= \int_{V'} \frac{\rho(r')}{4\pi\varepsilon_0}\left(-\nabla\frac{1}{|\vec r-\vec r'|}\right)dV'$$

$$= -\nabla\left[\int_{V'} \frac{\rho(r')}{4\pi\varepsilon_0}\frac{1}{|\vec r-\vec r'|}\,dV'\right]$$

$$\varphi = \int_{V'} \frac{\rho(r')}{4\pi\varepsilon_0}\frac{1}{|\vec r-\vec r'|}\,dV'$$ 电位定义式 $\boxed{\vec E(\vec r) = -\nabla\varphi}$

① 由性质 $\nabla\times\nabla u=0$ $\therefore \nabla\times\vec E(\vec r)=0$ 静电场无旋

② $\int_P^{P_0}\vec E\cdot d\vec l = \int_P^{P_0} -d\varphi = \varphi(P)-\varphi(P_0)$

$\vec E = -\nabla\varphi = -\left[\frac{\partial\varphi}{\partial x}\vec e_x + \frac{\partial\varphi}{\partial y}\vec e_y + \frac{\partial\varphi}{\partial z}\vec e_z\right]$

P  P_0

只与起点、终点电压差有关

$d\vec l = dx\,\vec e_x + dy\,\vec e_y + dz\,\vec e_z$

$\vec E\cdot d\vec l = -\left[\frac{\partial\varphi}{\partial x}dx + \frac{\partial\varphi}{\partial y}dy + \frac{\partial\varphi}{\partial z}dz\right]$ 全微分 $= -d\varphi$

若令 $\varphi(P_0)=0$ 则 $\varphi(P) = \int_P^{P_0}\vec E\cdot d\vec l$

③ $\nabla\cdot\vec E = \frac{\rho}{\varepsilon_0} \Rightarrow \nabla\cdot\nabla\varphi = -\frac{\rho}{\varepsilon_0}$ 即 $\nabla^2\varphi = -\frac{\rho}{\varepsilon_0}$

若 $\rho=0 \Rightarrow \nabla^2\varphi=0$

φ 是调和函数

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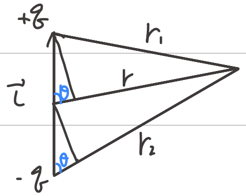
电偶极子:

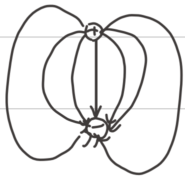


$r >> |l|$

$$\frac{1}{r_1}-\frac{1}{r_2}=\frac{r_2-r_1}{r_1 r_2}=\frac{l\cos\theta}{r^2-\frac{l^2}{4}\cos^2\theta}=\frac{l\cos\theta}{r^2}$$

$$r_1 = r-\frac{l}{2}\cos\theta$$
$$r_2 = r+\frac{l}{2}\cos\theta$$

电偶极矩 $\vec{p}=q\cdot\vec{l}$



$$\varphi=\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$
$$=\frac{q}{4\pi\varepsilon_0}\,\frac{l\cos\theta\cdot r}{r^3}=\frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}$$

$$E=-\nabla\varphi=\frac{|p|}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\vec{e}_r+\sin\theta\,\vec{e}_\theta\right) \qquad E\propto\frac{1}{r^3}$$

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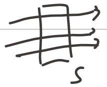
恒定电流与恒定电场

电流I



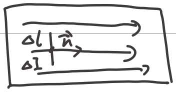
$I=\frac{\Delta q}{\Delta t}$

电流密度: $\vec{J}$

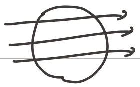
$\Delta S \perp \vec{n}$

$\vec{J}=\lim_{\Delta S\to 0}\frac{\Delta I}{\Delta S}\vec{n}$ $\boxed{I=\int_S \vec{J}\cdot d\vec{S}}$

面电流密度: $\vec{J}_S=\lim_{\Delta l\to 0}\frac{\Delta I}{\Delta l}\vec{n}$



2. 电流连续性方程



电流连续 ⟺ 电荷守恒

$\oint_S \vec{J}\cdot d\vec{S}=I_{流出}=-\frac{dq}{dt}$ 时间

$=-\frac{d}{dt}\int_V \rho\, dV$ 空间 $=-\int_V \frac{\partial\rho}{\partial t}dV$

即 $\boxed{\oint_S \vec{J}\cdot d\vec{S}=-\int_V\frac{\partial\rho}{\partial t}dV}$ 电流连续性方程积分形式

$\int_V \nabla\cdot\vec{J}\, dV=-\int_V\frac{\partial\rho}{\partial t}dV \Rightarrow \boxed{\nabla\cdot\vec{J}=-\frac{\partial\rho}{\partial t}}$ 微分形式

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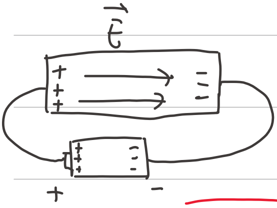
3. 恒定电流：$\vec{J}$ 不随时间变化.



$\vec{E}$：恒定电场（稳恒）　与静电场的区别.

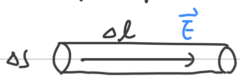
①电荷运动 ②在导体内

场方程 $\begin{cases} \boxed{\nabla \times \vec{E} = 0} & \text{旋度仍为0} \\ \boxed{\nabla \cdot \vec{J} = 0} & \text{电流密度散度为0} \end{cases}$ $\Rightarrow$ 积分形式：$\begin{cases} \boxed{\oint \vec{E} \cdot d\vec{l} = 0} \\ \boxed{\oint_S \vec{J} \cdot d\vec{S} = 0} \end{cases}$

1. 欧姆定律　$U = IR$

微分形式：$\vec{J} = \sigma \vec{E}$　σ：电导率

2. 焦耳定律　$P = UI$



ΔI . ΔU

$$\Delta P = \Delta U \cdot \Delta I = \vec{E} \cdot \Delta l \cdot \vec{J} \cdot \Delta S = \vec{E} \cdot \vec{J} \cdot \Delta V$$

$\therefore \dfrac{\Delta P}{\Delta V} = \vec{E} \cdot \vec{J}$　电功率体密度 $p = \lim\limits_{\Delta V \to 0} \dfrac{\Delta P}{\Delta V} = \vec{E} \cdot \vec{J}$

微分形式：$\boxed{p = \vec{E} \cdot \vec{J}}$

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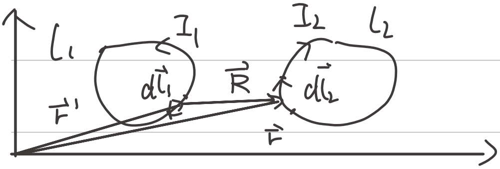
安培定律:



电流元 Idl

体电流. $\vec{J}dV$ $B(\vec{r})=\frac{\mu_0}{4\pi}\int_V\frac{\vec{J}\times\vec{e}_R}{R^2}dV$

面电流. $\vec{J}_s dS$ $B(\vec{r})=\frac{\mu_0}{4\pi}\int_S\frac{\vec{J}_s\times\vec{e}_R}{R^2}dS$

$$dF_{12}=\frac{\mu_0}{4\pi}\frac{I_2 dl_2\times(I_1 dl_1\times\vec{e}_R)}{|R|^2}$$

回路 l_1 整体对 $I_2 d\vec{l}_2$ 的作用力:

$$dF_2=I_2 dl_2\times\left(\underline{\frac{\mu_0}{4\pi}\oint_{l_1}\frac{I_1 dl_1\times\vec{e}_R}{|R|^2}}\right) = I_2 dl_2\times\vec{B}$$

磁感应强度

$$\boxed{\vec{B}(\vec{r})=\frac{\mu_0}{4\pi}\oint_{l_1}\frac{I_1 dl\times\vec{e}_R}{|R|^2}}$$ 毕奥-萨伐尔定律

$$\vec{F}_2=\oint_{l_2}dF_2=\oint_{l_2}I_2 dl_2\times\vec{B}=\oint_{l_2}\oint_{l_1}\frac{\mu_0}{4\pi}\frac{I_2 d\vec{l}_2\times(I_1 d\vec{l}_1\times\vec{e}_R)}{|R|^2}$$ 安培定律.

洛伦兹力: $\vec{F}=q\vec{v}\times\vec{B}$ 电磁力: $\vec{F}=q(\vec{E}+\vec{v}\times\vec{B})$

静磁场的场方程.

① $\dfrac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} = -\nabla\dfrac{1}{|\vec{r}-\vec{r}'|}$

② $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$.

③ $\nabla\times(u\vec{A}) = \nabla u\times\vec{A} + u\nabla\times\vec{A}$

$\vec{A}=\vec{J}$ $\quad u=\dfrac{1}{|\vec{r}-\vec{r}'|}$

④ $\nabla\cdot(\nabla\times\vec{A})=0$.

1. 磁通量连续性定理:

磁通量 $\Phi = \int_S \vec{B}\cdot d\vec{S}$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int_{V'}\frac{\vec{J}(\vec{r}')\times\vec{e}_R}{R^2}dV' \qquad \vec{R}=\vec{r}-\vec{r}'$$

代入

$$= \frac{\mu_0}{4\pi}\int_{V'}\frac{\vec{J}(\vec{r}')\times\vec{R}}{R^3}dV' = \frac{\mu_0}{4\pi}\int_{V'}\frac{\vec{J}\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}dV'$$

$$\overset{①}{=} \frac{\mu_0}{4\pi}\int_{V'}\vec{J}(\vec{r}')\times\left(-\nabla\frac{1}{|\vec{r}-\vec{r}'|}\right)dV'$$

$$\overset{②}{=} \frac{\mu_0}{4\pi}\int_{V'}\left[\nabla\frac{1}{|\vec{r}-\vec{r}'|}\times\vec{J}(\vec{r}')\right]dV'$$

$$\overset{③}{=} \frac{\mu_0}{4\pi}\int_{V'}\left[\nabla\times\frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|} - \frac{1}{|\vec{r}-\vec{r}'|}\overset{=0}{\boxed{\nabla\times\vec{J}(\vec{r}')}}\right]dV'$$

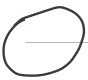
$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\nabla\times\int_{V'}\frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}dV'$$

④

$$\boxed{\begin{cases}\nabla\cdot\vec{B}=0\\ \int_V\nabla\cdot\vec{B}\,dV = \oint_S\vec{B}\cdot d\vec{S}=0\end{cases}}$$

析:
① 散度为0 磁场为无源场 旋涡场
② 不存在磁荷

2. 安培环路定理:

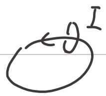


$$\boxed{\oint_l\vec{B}\cdot d\vec{l} = \mu_0 I}$$

$$\int_S(\nabla\times\vec{B})\cdot d\vec{S} = \mu_0\int_S\vec{J}\cdot d\vec{S}$$

$$\boxed{\nabla\times\vec{B} = \mu_0\vec{J}}$$

恒定电流与静磁场是交链的.



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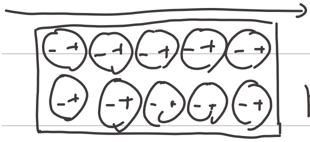
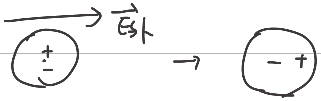
电介质的极化

电介质 = 绝缘体



$\vec E_{外}$

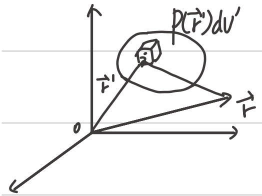
附加电场



$\vec E_{外}$

电偶极矩: $\vec p$

极化强度 $\vec P = \lim\limits_{\Delta V \to 0} \frac{\sum \vec p}{\Delta V}$ (电矩体密度)



$$\varphi = \int_{V'} \frac{\vec P(\vec r')\,dV' \cdot (\vec r - \vec r')}{4\pi\varepsilon_0 |\vec r - \vec r'|^3}$$

$$\varphi = \frac{\vec p \cdot \vec r}{4\pi\varepsilon_0 r^3}$$

$$= \int_{V'} \frac{\vec P \cdot (\vec r - \vec r')}{4\pi\varepsilon_0 |\vec r - \vec r'|^3} dV' \overset{①}{=} \frac{1}{4\pi\varepsilon_0}\int_{V'} \vec P(\vec r') \cdot \nabla' \frac{1}{|\vec r - \vec r'|} dV'$$

$$\overset{②}{=} \frac{1}{4\pi\varepsilon_0}\int_{V'} \left[\nabla' \cdot \frac{\vec P(\vec r')}{|\vec r - \vec r'|} - \frac{1}{|\vec r - \vec r'|} \nabla' \cdot \vec P(\vec r')\right] dV'$$

$$\overset{③}{=} \frac{1}{4\pi\varepsilon_0}\oint_S \frac{\overbrace{\vec P(\vec r')\cdot \vec n}^{\rho_s}}{|\vec r - \vec r'|} dS + \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{\overbrace{-\nabla' \cdot \vec P(\vec r')}^{\rho_v}}{|\vec r - \vec r'|} dV'$$

$$\overset{④}{=} \varphi_s + \varphi_v$$

即: $\varphi = \varphi_s + \varphi_v$ (束缚电荷)

$\rho_s = \vec P(\vec r) \cdot \vec n$ 极化电荷

$\rho_v = -\nabla \cdot \vec P(\vec r)$ 总量为0

(外加电荷 自由电荷) 总量可以不为0

① $\frac{\vec r - \vec r'}{|\vec r - \vec r'|^3} = \nabla' \frac{1}{|\vec r - \vec r'|}$ (对源点求)

② $\nabla' \cdot (u\vec A) = u\nabla' \cdot \vec A + \vec A \cdot \nabla' u$

令 $\vec A = \vec P(\vec r')$　$u = \frac{1}{|\vec r - \vec r'|}$

③ $\int_V \nabla \cdot \vec A\, dV = \oint \vec A \cdot d\vec S$

④ $\varphi_s = \int_S \frac{\rho_s}{|\vec r - \vec r'|} dS \cdot \frac{1}{4\pi\varepsilon_0}$

$\varphi_v = \int_V \frac{\rho_v}{|\vec r - \vec r'|} dV \quad \frac{1}{4\pi\varepsilon_0}$

介质中的场方程：

在真空中：

$$\begin{cases}\nabla\times\vec{E}=0\\ \nabla\cdot\vec{E}=\dfrac{\rho}{\varepsilon_0}\end{cases}$$

介质中：

$$\begin{cases}\nabla\times\vec{E}=0\\ \nabla\cdot\vec{D}=\rho\end{cases}\qquad\begin{cases}\oint_l\vec{E}\cdot d\vec{l}=0\\ \oint_S\vec{D}\cdot d\vec{S}=Q\end{cases}$$

介电常数：

$\vec{P}$与$\vec{E}$的关系 组成关系

各向同性 线性介质

$\vec{P}=\varepsilon_0\chi_e\vec{E}$ χ_e：极化率

$$\nabla\cdot\vec{E}=\frac{\rho_外+\rho_p}{\varepsilon_0}\qquad \rho_p=-\nabla\cdot\vec{P}$$

$$\nabla\cdot(\varepsilon_0\vec{E}+\vec{P})=\rho_外$$

电位移矢量 $\vec{D}=\varepsilon_0\vec{E}+\vec{P}$

$$\vec{D}=\varepsilon_0\vec{E}+\varepsilon_0\chi_e\vec{E}=\varepsilon_0(1+\chi_e)\vec{E}$$

$$=\varepsilon_0\varepsilon_r\vec{E}=\varepsilon\vec{E}$$

ε_r：相对介电常数

ε：介电常数

真空中：$\nabla^2\varphi=-\dfrac{\rho}{\varepsilon_0}$

介质中：$\nabla^2\varphi=-\dfrac{\rho}{\varepsilon}$

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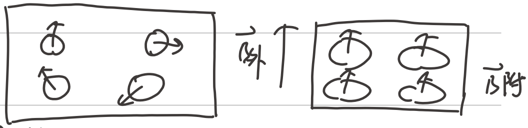
磁化现象：



分类：
- 抗磁性：电磁感应
- 顺磁性：分子磁矩 $\vec m$ 与外磁场一致
- 铁磁性 磁畴

磁化强度：$\vec M = \lim_{\Delta V\to 0}\frac{\Delta \vec m}{\Delta V}$ 磁矩体密度

磁化体电流：$\boxed{\vec J_m = \nabla\times\vec M}$ 　　磁化面电流：$\boxed{\vec J_{ms} = \vec M\times\vec n}$

磁场强度 $\vec H$

真空中：$\nabla\times\vec B = \mu_0\vec J$

在介质中：$\nabla\times\vec B = \mu_0(\vec J+\vec J_m) = \mu_0(\vec J+\nabla\times\vec M)$

$$\nabla\times(\underbrace{\frac{\vec B}{\mu_0}-\vec M}_{\text{磁场强度}\vec H}) = \vec J \qquad 令\ \vec H = \frac{\vec B}{\mu_0}-\vec M \iff \boxed{\vec B=\mu_0(\vec H+\vec M)}$$

$$\boxed{\begin{cases}\nabla\times\vec H=\vec J\\ \oint_L \vec H\cdot d\vec l = I\end{cases}}$$

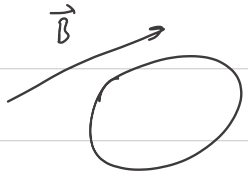
ie: $\oint_L \vec H\cdot d\vec l = \int_S \nabla\times\vec H\cdot d\vec S = \int_S \vec J\cdot d\vec S = I$

各向同性的均匀介质中

$\vec M = \chi_m\vec H$ 　　χ_m：磁化率.

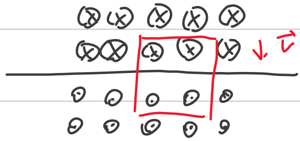
$$\vec B = \mu_0(\vec H+\vec M) = \mu_0(\underbrace{1+\chi_m}_{\mu_r\ \text{相对磁导率}})\vec H = \underbrace{\mu_0\mu_r}_{\mu\ \text{磁导率}}\vec H = \mu\vec H$$

法拉第电磁感应定律



感应电动势 $\mathcal{E} = -\frac{d\Phi}{dt}$

$$= -\lim_{\Delta t \to 0} \frac{\Delta \Phi}{\Delta t}$$



① $\vec{B}$ 随时间变化

② 线圈运动

$$\Delta\Phi = \Phi(t+\Delta t) - \Phi(t)$$

$$= \int_{S_b} \vec{B}(t+\Delta t)\cdot d\vec{S} - \int_{S_a} \vec{B}(t)\cdot d\vec{S}$$

(证明略)

$$\boxed{\mathcal{E} = \underbrace{\int_S -\frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}}_{\text{感生}} + \underbrace{\int_l (\vec{v}\times\vec{B})\cdot d\vec{l}}_{\text{动生}}}$$

感应电场 $\vec{E}_{ind}$，静电场 $\vec{E}_c$　　$\oint \vec{E}_c \cdot d\vec{l} = 0$　$\nabla\times\vec{E}_c = 0$

$$\oint \vec{E}_{ind}\cdot d\vec{l} = \int_S -\frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$$

总电场 $\vec{E} = \vec{E}_{ind} + \vec{E}_c$

$$\boxed{\oint \vec{E}\cdot d\vec{l} = \int_S -\frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}}$$ 积分形式

$$\boxed{\nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$ 微分形式

日期: /

1. 位移电流

电流连续性方程 $\nabla\cdot\vec{J}=-\frac{\partial\rho}{\partial t}$

静态场安培环路定理: $\nabla\times\vec{H}=\vec{J}$

$\nabla\cdot(\nabla\times\vec{H})=-\frac{\partial\rho}{\partial t}$

$\boxed{\nabla\cdot(\nabla\times\vec{H})=0}=-\frac{\partial\rho}{\partial t}+\frac{\partial\rho}{\partial t}$ 又: $\nabla\cdot\vec{D}=\rho$

$$\begin{aligned}\nabla\cdot(\nabla\times\vec{H})&=\nabla\cdot\vec{J}+\frac{\partial\rho}{\partial t}\\&=\nabla\cdot\vec{J}+\frac{\partial(\nabla\cdot\vec{D})}{\partial t}\\&=\nabla\cdot\left(\vec{J}+\frac{\partial\vec{D}}{\partial t}\right)\end{aligned}$$

$\therefore\ \nabla\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t}$ 位移电流面密度 $\vec{J}_d=\frac{\partial\vec{D}}{\partial t}$ (不是电荷移动产生的)

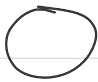
动态场中: $\boxed{\nabla\times\vec{H}=\underbrace{\vec{J}}_{\text{电荷移动产生}}+\frac{\partial\vec{D}}{\partial t}}$ 时变电场能产生磁场.

2. 全电流密度: $\vec{J}_t$

传导电流 导体中 $\vec{J}_c=\sigma\vec{E}$

运流电流 气体中/真空中 $\vec{J}_v$

位移电流 $\vec{J}_d$



封闭曲面全电流始终为零

$\vec{J}_t=\vec{J}_c+\vec{J}_v+\vec{J}_d$ $\nabla\cdot\vec{J}_t=\nabla\cdot(\nabla\times\vec{H})=0$

$$\int\nabla\cdot\vec{J}_t\,dV=\oint_S\vec{J}_t\cdot d\vec{S}=\boxed{I_t=I_c+I_v+I_d=0}$$

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1. 麦克斯韦方程组

微分形式:

① $\nabla\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t}$ 全电流定律.

② $\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$ 法拉第电磁感应定律.

③ $\nabla\cdot\vec{B}=0$ 磁通连续性定律.

④ $\nabla\cdot\vec{D}=\rho$ 高斯定理

积分形式

① $\oint_l\vec{H}\cdot d\vec{l}=\int_S(\vec{J}+\frac{\partial\vec{D}}{\partial t})\,d\vec{S}$

② $\oint_l\vec{E}\cdot d\vec{l}=\int_S-\frac{\partial\vec{B}}{\partial t}\,d\vec{S}$

③ $\oint_S\vec{B}\cdot d\vec{S}=0$

④ $\oint_S\vec{D}\,d\vec{S}=\int_V\rho\,dV=q$

对①两边取散度: $\nabla\cdot\vec{J}=-\frac{\partial\rho}{\partial t}$ 电流连续性方程

对②: $-\frac{\partial}{\partial t}(\nabla\cdot\vec{B})=0$ 在某一时刻 $\nabla\cdot\vec{B}=0$(条件) ⇒ 导出③

2. 本构关系:

① $\vec{D}=\varepsilon_0\vec{E}+\vec{P}$　　$\vec{D}=\varepsilon\vec{E}$

② $\vec{B}=\mu_0(\vec{H}+\vec{M})$.　　$\vec{B}=\mu\vec{H}$

③ $\vec{J}=\sigma\vec{E}$

3. 洛伦兹力

$\vec{F}=q(\vec{E}+\vec{v}\times\vec{B})$

洛伦兹力密度

$$f=\rho(\vec{E}+\vec{v}\times\vec{B})$$

$$=\rho\vec{E}+\rho\cdot\vec{v}\times\vec{B}\quad \text{又}\because\vec{J}=\rho\cdot\vec{v}$$

$$=\rho\vec{E}+\vec{J}\times\vec{B}$$

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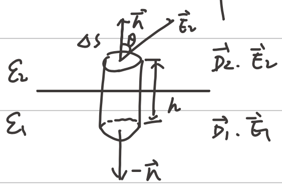
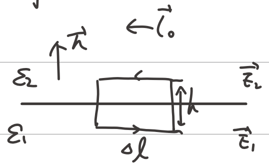
静电场的边界条件：场量在介质面上的变化规律



$$\begin{cases} \oint \vec{D}\cdot d\vec{S} = Q \\ \oint \vec{E}\cdot d\vec{l} = 0 \end{cases}$$



$$\vec{E}_2\cdot\Delta l\,\vec{l}_0 - \vec{E}_1\Delta l\,\vec{l}_0 = 0$$

$$(\vec{E}_2-\vec{E}_1)\cdot\vec{l}_0 = 0$$

$$(\vec{E}_2-\vec{E}_1)\perp\vec{l}_0$$

$$(\vec{E}_2-\vec{E}_1)\parallel\vec{n}$$

$$\boxed{\begin{cases} \vec{n}\times(\vec{E}_2-\vec{E}_1)=0 \\ E_{1t}=E_{2t} \quad \text{(切向分量)} \end{cases}}$$

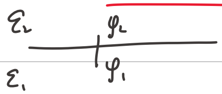
$$\vec{D}_2\cdot(\Delta S\cdot\vec{n}) - \vec{D}_1\cdot(\Delta S\cdot\vec{n}) = \rho_s\cdot\Delta S$$

$$\boxed{\begin{cases} \vec{n}\cdot(\vec{D}_2-\vec{D}_1)=\rho_s \\ D_{2n}-D_{1n}=\rho_s \quad \text{(法向方向)} \end{cases}}$$

$$\varepsilon_2E_{2n} - \varepsilon_1E_{1n} = \rho_s$$

$$E_{2n} = |E_2|\cos\theta = -|\nabla\varphi|\cos\theta = -\frac{\partial\varphi}{\partial n}$$

$$-\varepsilon_2\frac{\partial\varphi_2}{\partial n} + \varepsilon_1\frac{\partial\varphi_1}{\partial n} = \rho_s$$

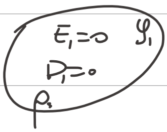


$$\varphi_2-\varphi_1 = \int_2^1 \vec{E}\cdot d\vec{l} = 0$$

2和1相距很小

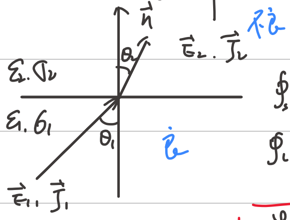
$$\boxed{\begin{cases} \varphi_1=\varphi_2 \quad \text{(电位连续)} \\ \varepsilon_1\frac{\partial\varphi}{\partial n} - \varepsilon_2\frac{\partial\varphi}{\partial n} = \rho_s \end{cases}}$$

特殊：导体表面：



$$\boxed{\begin{matrix} E_{2t}=0 \\ D_{2n}=\rho_s \\ \varphi_2=\varphi_1 \end{matrix}}$$

恒定电场的边界条件。



$\oint_S \vec J\cdot d\vec S = 0 \Rightarrow \boxed{\begin{array}{l} J_{1n} = J_{2n} \\ \vec n\cdot(\vec J_1-\vec J_2)=0 \end{array}}$ 法向 $\sigma_1 E_{1n} = \sigma_2 E_{2n}$

$\oint_l \vec E\cdot d\vec l = 0 \Rightarrow \boxed{\begin{array}{l} E_{1t} = E_{2t} \\ \vec n\times(\vec E_1-\vec E_2)=0 \end{array}}$ 切向 $\dfrac{J_{1t}}{\sigma_1} = \dfrac{J_{2t}}{\sigma_2}$

$$\boxed{\begin{cases} \varphi_1 = \varphi_2 \\ \sigma_1 \dfrac{\partial \varphi_1}{\partial n} = \sigma_2 \dfrac{\partial \varphi_2}{\partial n} \end{cases}}$$ 电压连续

$$\begin{array}{l} |J_1|\cos\theta_1 = |J_2|\cos\theta_2 \\ \sigma_1|\vec E_1|\cos\theta_1 = \sigma_2|\vec E_2|\cos\theta_2 \\ |\vec E_1|\sin\theta_1 = |\vec E_2|\sin\theta_2 \end{array} \Rightarrow \boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\sigma_1}{\sigma_2}}$$

$\sigma_1 \gg \sigma_2 \Rightarrow \theta_1 \gg \theta_2$

$\theta_1 \neq \frac{\pi}{2} \Rightarrow \theta_2 \approx 0$

面电荷积累.

$$\rho_s = (D_{2n} - D_{1n})$$
$$= \varepsilon_2 \vec E_{2n} - \varepsilon_1 \vec E_{1n}$$
$$= \varepsilon_2 \frac{\vec J_{2n}}{\sigma_2} - \varepsilon_1 \frac{\vec J_{1n}}{\sigma_1} \qquad \vec J_{2n} = \vec J_{1n}$$
$$\boxed{\rho_s = \vec J_n\left(\frac{\varepsilon_2}{\sigma_2} - \frac{\varepsilon_1}{\sigma_1}\right)}$$

若 $\rho_s = 0 \Rightarrow$
① $J_n = 0$ 有一个媒质是电介质
② $\dfrac{\varepsilon_2}{\sigma_2} = \dfrac{\varepsilon_1}{\sigma_1}$ （较难实现）

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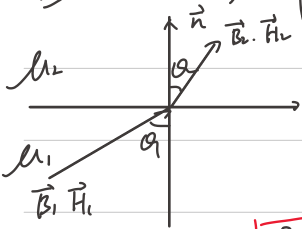
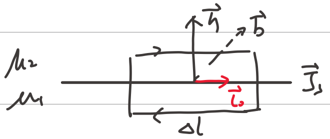
静磁场的边界条件.



t是面的法向量



面电流

$\vec{H}_2\cdot\Delta l\,\vec{t}_0 - \vec{H}_1\Delta l\,\vec{t}_0 = \vec{J}_s\cdot\Delta l\cdot\vec{t}$

$\oint\vec{B}\cdot d\vec{S}=0 \Rightarrow \boxed{\begin{cases}B_{1n}=B_{2n}\\ \vec{n}\cdot(\vec{B}_1-\vec{B}_2)=0\end{cases}}$

$(\vec{H}_2-\vec{H}_1)\,\vec{t}_0 = \vec{J}_s\,\vec{t}$

$\overset{①}{\Rightarrow}(\vec{t}\times\vec{n})\cdot(\vec{H}_2-\vec{H}_1)=\vec{J}_s\cdot\vec{t}.$

$\oint\vec{H}\cdot d\vec{l} = \int_S\vec{J}\cdot d\vec{S}$

$\overset{②}{\Rightarrow}\ \vec{n}\times(\vec{H}_2-\vec{H}_1)\cdot\vec{t} = \vec{J}_s\cdot\vec{t}$

① $\vec{t}_0 = \vec{t}\times\vec{n}$

② $\vec{A}\times\vec{B}\cdot\vec{C} = \vec{B}\times\vec{C}\cdot\vec{A}$

③ $|\vec{n}\times\vec{A}| = A_t$

$\Rightarrow \boxed{\begin{aligned}&\vec{n}\times(\vec{H}_2-\vec{H}_1)=\vec{J}_s\\ &H_{2t}-H_{1t}=|\vec{J}_s|\end{aligned}}$

$\vec{A} = A_n\vec{n}+A_t\vec{t}$

$\vec{n}\times\vec{A} = \vec{n}\times(A_n\vec{n}+A_t\vec{t})$ $\quad \underline{|\vec{n}\times\vec{A}| = A_t}$

$= A_t\cdot(\vec{n}\times\vec{t})$

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电磁场的边界条件:

$$\begin{cases} \vec{n}\cdot(\vec{D}_2-\vec{D}_1)=\rho_s \\ \vec{n}\times(\vec{E}_2-\vec{E}_1)=0 \\ \vec{n}\cdot(\vec{B}_2-\vec{B}_1)=0 \\ \vec{n}\times(\vec{H}_2-\vec{H}_1)=\vec{J}_s \end{cases}$$

既适用于静态场/也适用于时变场.

在理想介质表面 $(\sigma=0)$

$\vec{J}_s=0,\ \rho_s=0$ 代入左式

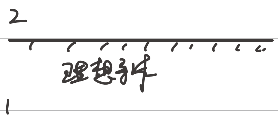
$$\begin{cases} \vec{n}\cdot(\vec{D}_2-\vec{D}_1)=0 \\ \vec{n}\times(\vec{E}_2-\vec{E}_1)=0 \\ \vec{n}\cdot(\vec{B}_2-\vec{B}_1)=0 \\ \vec{n}\times(\vec{H}_2-\vec{H}_1)=0 \end{cases}$$

理想导体 $(\sigma\to\infty)$ $\vec{J}=\sigma\vec{E}\to+\infty$

内部的电场.磁场均为0

导体表面外侧

$$\begin{cases} \vec{n}\cdot\vec{D}=\rho_s \\ \vec{n}\times\vec{E}=0 \\ \vec{n}\cdot\vec{B}=0 \\ \vec{n}\times\vec{H}=\vec{J}_s \end{cases}$$



在理想导体表面. 电场线是垂直于理想导体表面

磁场线 平行于 - —

导体系统的电容

$\varphi_{ij} = p_{ij} q_j$

j在i处产生电位

$\varphi_i = \sum_j \varphi_{ij} = \sum_j p_{ij} q_j$

p_{ij}: 电位系数

$$\varphi_1 = p_{11} q_{11} + p_{12} q_{12} + p_{13} q_{13} + \cdots + p_{1n} q_{1n}$$
$$\varphi_2 = p_{21} q_{21} + p_{22} q_{22} + \cdots + p_{2n} q_{2n}$$
$$\vdots$$
$$\varphi_n = p_{n1} q_{n1} + p_{n2} q_{n2} + \cdots + p_{nn} q_{nn}$$

$[\varphi] = [p][q]$

电位系数矩阵.

性质. $p_{ii} > p_{ij}$ $\quad p_{ij} = p_{ji}$

$[q] = [p]^{-1}[\varphi]$ $\quad \beta$: 电容系数

$$q_1 = \beta_{11}\varphi_1 + \beta_{12}\varphi_2 + \cdots + \beta_{1n}\varphi_n$$
$$\vdots$$
$$q_n = \beta_{n1}\varphi_1 + \beta_{n2}\varphi_2 + \cdots \beta_{nn}\varphi_n$$

$$q_1 = (\beta_{11} + \beta_{12} + \cdots + \beta_{1n})\varphi_1 - \beta_{12}(\varphi_1 - \varphi_2) - \cdots - \beta_{1n}(\varphi_1 - \varphi_n)$$

$C_{ii} = \sum_j \beta_{ij}$ $\quad C_{ij} = -\beta_{ij}$

自部分电容 $\quad$ 互部分电容

(与大地之间的电容)

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带电系统的能量:

$\cdot\, q$ $\qquad$ $\cdot\, q$

从无到有建立带电系统所需能量

αq_1, $\alpha\varphi_1$ $\qquad$ αq_2, $\alpha\varphi_2$

αq_4, $\alpha\varphi_4$ $\qquad$ αq_3, $\alpha\varphi_3$

$\alpha=0 \rightarrow \alpha\uparrow \rightarrow \alpha=1$

$\alpha \rightarrow \alpha+d\alpha$

将$q_i\, d\alpha$移至$\alpha\varphi_i$处需做功

点电荷: $W_e=\int_0^1 \sum_i \alpha\varphi_i q_i\, d\alpha$

$= \sum_i q_i\varphi_i \int_0^1 \alpha\, d\alpha = \frac{1}{2}\sum_i q_i\varphi_i$

分布电荷: $W_e = \frac{1}{2}\int_V \rho(\vec{r})\varphi(\vec{r})\, dV$

能量密度. $W_e = \frac{1}{2}\int_V \rho\varphi\, dV$

$= \frac{1}{2}\int_V (\nabla\cdot\vec{D})\varphi\, dV$

$= \frac{1}{2}\int_V [\nabla\cdot(\varphi\vec{D}) - \vec{D}\cdot\nabla\varphi]\, dV$

$= \frac{1}{2}\oint_S \varphi\vec{D}\cdot d\vec{S} + \frac{1}{2}\int_V \vec{E}\cdot\vec{D}\, dV$

$= \frac{1}{2}\int_V \boxed{\vec{E}\cdot\vec{D}}\, dV$

① $\nabla\cdot\vec{D}=\rho$

② $\nabla\cdot(u\vec{A}) = \boxed{u\nabla\cdot\vec{A}} + \vec{A}\cdot\nabla u$

$\vec{A}=\vec{D}$, $u=\varphi$.

③ $\int_V \nabla\cdot\vec{A}\, dV = \oint_S \vec{A}\cdot d\vec{S}$

$-\nabla\varphi = \vec{D}$

④ R

$\varphi \propto \frac{1}{R}$ $\quad \vec{D}\propto\frac{1}{R^2}$ $\quad \int dS \propto R^2$

$\oint_S \varphi\vec{D}\cdot d\vec{S} \propto \frac{1}{R}$ $\quad$ R趋大 $\rightarrow 0$.

能量密度: $w_e = \frac{1}{2}\vec{E}\cdot\vec{D}$

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矢量磁位(磁矢位) $\vec{A}$: $\boxed{\vec{B}=\nabla\times\vec{A}}$

φ: $\vec{E}=-\nabla\varphi$

不唯一: $\vec{A}'=\vec{A}+\nabla\phi$

$\varphi'=\varphi+c$

$$\nabla\times\vec{A}'=\nabla\times(\vec{A}+\nabla\phi)=\nabla\times\vec{A}+\underline{\overset{\to 0}{\nabla\times(\nabla\phi)}}=\vec{B}$$

指定一个零电位点

指定 $\boxed{\nabla\cdot\vec{A}=0 \quad \text{(库仑规范)}}$

场方程: $\boxed{\nabla^2\vec{A}=-\mu_0\vec{J}}$

场方程: $\nabla^2\varphi=-\frac{\rho}{\varepsilon_0}$

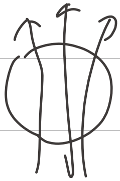
(泊松方程)

体分布电流: $\vec{A}(\vec{r})=\frac{\mu_0}{4\pi}\int_{V'}\frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}dV'$

面分布电流: $\vec{A}(\vec{r})=\frac{\mu_0}{4\pi}\int_{S'}\frac{\vec{J}_S(\vec{r}')}{|\vec{r}-\vec{r}'|}dS'$

$\varphi(\vec{r})=\frac{1}{4\pi\varepsilon_0}\int_{V'}\frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}dV'$

线分布电流: $\vec{A}(\vec{r})=\frac{\mu_0 I}{4\pi}\int_{l'}\frac{1}{|\vec{r}-\vec{r}'|}d\vec{l}'$



磁通量: $\Phi=\int_S\vec{B}\cdot d\vec{S}=\int_S(\nabla\times\vec{A})\cdot d\vec{S}$

$$\boxed{\Phi=\oint_l\vec{A}\cdot d\vec{l}}$$

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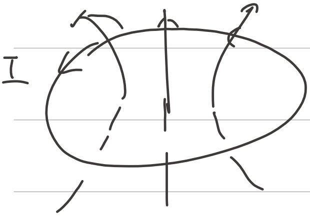
电感：自感与互感.

磁链：总磁通



I

N匝，每匝磁通为 Φ

$\Psi = N \cdot \Phi$

$$\boxed{L = \frac{\Phi}{I} \quad \text{自感}}$$

互感：M_{12} M_{21}

C₁回路形成的磁场在C₂上的磁通：φ_{12}

$$\boxed{M_{12} = \frac{\varphi_{12}}{I_1}, \quad M_{21} = \frac{\varphi_{21}}{I_2}}$$

$$\varphi_{12} = \int_{S_2} \vec{B}_1 \cdot d\vec{S}_2 = \int_{S_2} (\nabla \times \vec{A}_1) \cdot d\vec{S}_2$$

$$= \oint_{C_2} \vec{A}_1 \cdot d\vec{l}_2$$

$$\vec{A}_1 = \frac{\mu_0 I_1}{4\pi} \oint_{C_1} \frac{d\vec{l}_1}{|\vec{r}_2 - \vec{r}_1|}$$

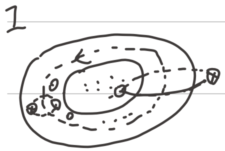
$$\boxed{\varphi_{12} = \frac{\mu_0 I_1}{4\pi} \oint_{C_1} \oint_{C_2} \frac{d\vec{l}_1 \cdot d\vec{l}_2}{|\vec{r}_2 - \vec{r}_1|}}$$ 纽曼公式

$$M_{12} = \frac{\varphi_{12}}{I_1} = \frac{\mu_0}{4\pi} \oint_{C_1} \oint_{C_2} \frac{d\vec{l}_1 \cdot d\vec{l}_2}{|\vec{r}_2 - \vec{r}_1|}$$ 与I_1、I_2都无关

$M_{21} = M_{12}$ 互易性

法一：定义式：$\varphi_{12} \to M_{12} = \frac{\varphi_{12}}{I_1}$

法二：纽曼公式



导体尺寸不可忽略时：

$L = L_{in} + L_{out}$

L_{in}：内自感 L_{out}：外自感

$$\frac{\text{导体外磁链}}{\text{总电流}} = L_{out}$$

$$\frac{\text{导体内磁链}}{\text{总电流}} = L_{in}$$

外自感磁场线与整个电流交链

内 部分 $N<1$

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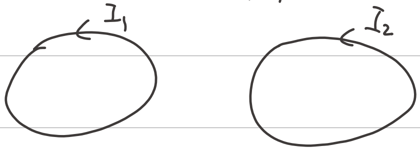
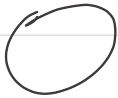
磁场能量:

从无到有建立磁场所需能量



法拉第电磁感应定律



磁通量 Φ

感应电动势 $\varepsilon = -\frac{d\psi}{dt}$

step 1: $i_1: 0 \to I_1$, 同时 $i_2 = 0$

step 2. $i_1 = I_1$. 同时 $i_2 = 0 \to I_2$

$L = \frac{\psi}{I}$ $\quad \psi_{11} = i_1 L_1 \quad d\psi_{11} = L_1 di_1$

$M_{12} = \frac{\psi_{12}}{I_1} \quad \psi_{12} = i_1 M_{12} \quad d\psi_{12} = M_{12} di_1$

$\underline{M_{12} = M_{21} = M}$

step 1. $\varepsilon_{11} = -\frac{d\psi_{11}}{dt} = -L_1 \frac{di_1}{dt}$

$\varepsilon_{12} = -\frac{d\psi_{12}}{dt} = -M \frac{di_1}{dt}$

$U_1 = -\varepsilon_{11} = L_1 \frac{di_1}{dt} \qquad U_2 = -\varepsilon_{12} = M \frac{di_1}{dt}$

当 $i_1 \to i_1 + di_1$ 时

$dW_1 = U_1 i_1 dt + U_2 i_2 dt = L_1 \frac{di_1}{dt} \cdot i_1 \cdot dt = L_1 i_1 di_1$

$$\boxed{W_1 = \int_0^{I_1} L_1 i_1 di_1 = \frac{1}{2} L I_1^2}$$

step 2: $\varepsilon_{21} = -\frac{d\psi_{21}}{dt} = -M \frac{di_2}{dt} \Rightarrow U_1 = M \frac{di_2}{dt}$

$\varepsilon_{22} = -\frac{d\psi_{22}}{dt} = -L_2 \frac{di_2}{dt} \qquad U_2 = L_2 \frac{di_2}{dt}$

当 $i_2 \to i_2 + di_2$ 时

$dW_2 = U_1 i_1 dt + U_2 i_2 dt = M I_1 di_2 + L_2 i di_2$

$$\boxed{W_2 = \int_0^{I_2} M I_1 di_2 + L_2 i di_2 = M I_1 I_2 + \frac{1}{2} L_2 I_2^2}$$

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$\Rightarrow W_m = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M I_1 I_2 = \frac{1}{2}I_1(L_1 I_1 + M I_2) + \frac{1}{2}I_2(L_2 I_2 + M I_1)$

$= \frac{1}{2}I_1\psi_1 + \frac{1}{2}I_2\psi_2$

推广: $W_m = \frac{1}{2}\sum_{i=1}^{n} I_i\psi_i$ 对比电场能量: $W_e = \frac{1}{2}\sum_i q_i\varphi_i$

$\psi_i = \int_{S_i}\vec{B}_i\cdot d\vec{S}_i = \int_{S_i}(\nabla\times\vec{A}_i)\cdot d\vec{S}_i$

$= \oint_{l_i}\vec{A}_i\cdot d\vec{l}_i$

$W_m = \frac{1}{2}\sum_{i=1}^{n}\oint_{l_i}\vec{A}_i\cdot\boxed{I_i\, d\vec{l}_i}$ 线电流向电流推广.

$W_m = \frac{1}{2}\sum_{i=1}^{n}\int_V \vec{A}_i\cdot\vec{J}_i\, dV_i$ 体

$\boxed{W_m = \frac{1}{2}\int_V \vec{A}\cdot\vec{J}\, dV}$ 对比电场. $W_e = \frac{1}{2}\int_V \rho\varphi\, dV$

磁能密度: w_m

① $\nabla\times\vec{H} = \vec{J}$

② $\nabla\cdot(\vec{A}\times\vec{H}) = \vec{H}\cdot(\nabla\times\vec{A}) - \vec{A}\cdot(\nabla\times\vec{H})$

$W_m = \frac{1}{2}\int_V \vec{A}\cdot\vec{J}\, dV$

$= \frac{1}{2}\int_V \vec{A}\cdot(\nabla\times\vec{H})\, dV$

$= \frac{1}{2}\int_V [\vec{H}\cdot(\nabla\times\vec{A}) - \nabla\cdot(\vec{A}\times\vec{H})]\, dV$

$= \frac{1}{2}\int_V \vec{H}\cdot\vec{B}\, dV - \underline{\frac{1}{2}\oint_S(\vec{A}\times\vec{H})\cdot d\vec{S}} \to 0$

$= \frac{1}{2}\int_V \boxed{\vec{H}\cdot\vec{B}}\, dV$

R

$\vec{A} = \frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r}')}{R}dV'$

$\vec{A}\sim\frac{1}{R}$

$\vec{H} = \frac{\vec{B}}{\mu}\sim\frac{1}{R^2}$

$\vec{B} = \frac{\mu_0}{4\pi}\int_{V'}\frac{\vec{J}(\vec{r}')\times\vec{e}_R}{R^2}dV'$

$|\vec{B}|\sim\frac{1}{R^2}$

$\oint_S d\vec{S}\sim R^2$

磁场能密度: $\boxed{w_m = \frac{1}{2}\vec{B}\cdot\vec{H}}$ $= \frac{1}{2}\mu\vec{H}\cdot\vec{H}$

$= \frac{1}{2}\mu|H|^2$ (各向同性)

与电场比较: $w_e = \frac{1}{2}\vec{E}\cdot\vec{D} = \frac{1}{2}\varepsilon|E|^2$

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静态场问题的分类:

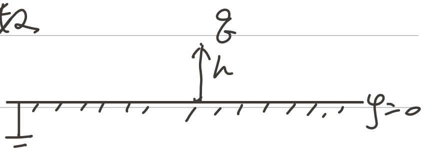
1. 分布型问题: 已知场源分布. 求整个空间中的场量.

$$\varphi=\frac{1}{4\pi\varepsilon_0}\int_{V'}\frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}dv' \qquad \vec{E}=-\nabla\varphi.$$

$$\vec{A}=\frac{\mu_0}{4\pi}\int_{V'}\frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}dv' \qquad \vec{B}=\nabla\times\vec{A}$$

先求位函数 → 场量

2. 边值型问题: 已知场源分布以及区域边界上的位函数.
求区域内的位函数



① 已知边界上的位函数值

$$\varphi(\vec{r})\big|_S=f(\vec{r})$$

② 已知边界上的位函数的方向导数

$$\frac{\partial\varphi(\vec{r})}{\partial n}\bigg|_S=g(\vec{r})$$

③ 边界 $S=S_1+S_2$. 已知 S_1 上位函数值. S_2 上位函数上的方向导数

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唯一性定理:

电荷分布. } $\Rightarrow$ 静电场是唯一确定的

边值条件

$\nabla\cdot\vec{E} = -\frac{\rho}{\varepsilon_0}$

只需证 $\varphi_1 - \varphi_2 = C$

证明: 反证法: 假设存在 φ_1. φ_2, 令 $\varphi = \varphi_1 - \varphi_2$

$$\begin{cases} \nabla^2\varphi_1 = -\frac{\rho}{\varepsilon} \\ \nabla^2\varphi_2 = -\frac{\rho}{\varepsilon} \end{cases} \Rightarrow \nabla^2\varphi = 0$$

$$\vec{E}_1 - \vec{E}_2 = -\nabla\varphi_1 - (-\nabla\varphi_2) = -\nabla(\varphi_1 - \varphi_2) = -\nabla C = 0$$

第一类: $\begin{cases} \varphi_1(\vec{r})|_s = f(\vec{r}) \\ \varphi_2(\vec{r})|_s = f(\vec{r}) \end{cases} \Rightarrow \varphi|_s = 0$

格林公式 令 $\psi = \varphi$

$$\int_V (\varphi\nabla^2\varphi + \nabla\varphi\cdot\nabla\varphi)dv = \oint_s \varphi\frac{\partial\varphi}{\partial n}ds$$

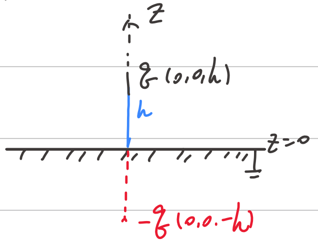
$$\int_V |\nabla\varphi|^2 dV = 0$$

$|\nabla\varphi| = 0 \quad \nabla\varphi = 0 \Rightarrow \varphi = C$

第二类 $\begin{cases} \frac{\partial\varphi_1}{\partial n}\Big|_s = g(\vec{r}) \\ \frac{\partial\varphi_2}{\partial n}\Big|_s = g(\vec{r}) \end{cases} \Rightarrow \frac{\partial\varphi}{\partial n}\Big|_s = 0$

1 平面镜像法:

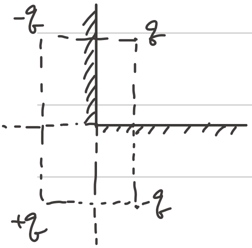
例.

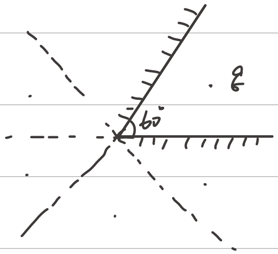


$$\begin{cases} z>0 & \nabla^2\varphi=0 \\ z=0, & \varphi=0\ (接地) \\ 无穷远处 & \varphi=0 \end{cases}$$

当 $z>0$ 时 $\varphi=\varphi_q+\varphi_{-q}$

$$=\frac{q}{4\pi\varepsilon_0 R_q}-\frac{q}{4\pi\varepsilon_0 R_{-q}}=\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{x^2+y^2+(z-h)^2}}-\frac{1}{\sqrt{x^2+y^2+(z+h)^2}}\right]$$





若夹角 $\theta=\frac{\pi}{n}$.

则镜像电荷数目为 $(2n-1)$ 个

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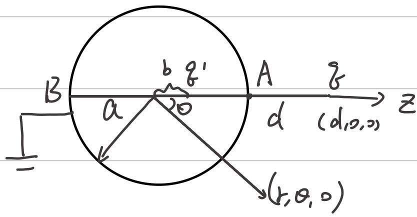
2、球面镜像法:

例:



$r>0 \quad \nabla^2\varphi=0$

$r=a. \quad \varphi=0$ (接地)

$r\to\infty. \quad \varphi=0$

A点: $\varphi_A = \dfrac{q}{4\pi\varepsilon_0(d-a)} + \dfrac{q'}{4\pi\varepsilon_0(a-b)} = 0$ 　　 q'与b为未知数

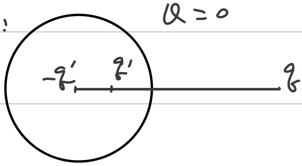
B点: $\varphi_B = \dfrac{q}{4\pi\varepsilon_0(d+a)} + \dfrac{q'}{4\pi\varepsilon_0(a+b)} = 0$

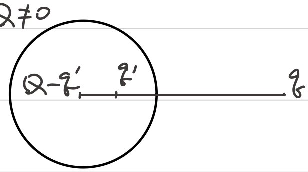
解得: $\begin{cases} b = \dfrac{a^2}{d} \\ q' = -\dfrac{a}{d}q \end{cases}$

$\varphi = \varphi_q + \varphi_{q'} = 0$

只求导体球外电位

若不接地: 　$Q=0$ 　　　　 $Q\neq 0$





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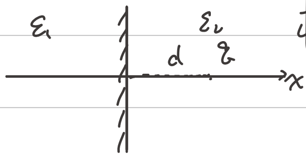
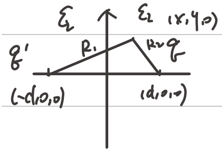
3. 介质平面镜像法:

例:



极化电荷

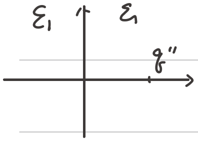
当 $x>0$ 时



$$\varphi_2=\frac{1}{4\pi\varepsilon_2}\left(\frac{q}{R_2}+\frac{q'}{R_1}\right)$$

$$=\frac{1}{4\pi\varepsilon_2}\left(\frac{q}{\sqrt{(x-d)^2+y^2}}+\frac{q'}{\sqrt{(x+d)^2+y^2}}\right)$$

当 $x<0$ 时.



q'': q与极化电荷的总的等效电荷

$$\varphi_1=\frac{1}{4\pi\varepsilon_1}\frac{q''}{\sqrt{(x-d)^2+y^2}}$$

由边界条件: $\begin{cases}\varphi_1=\varphi_2\\ \varepsilon_1\dfrac{\partial\varphi_1}{\partial n}=\varepsilon_2\dfrac{\partial\varphi_2}{\partial n}\end{cases}$　　n与x同向

代入: $\begin{cases}\dfrac{1}{\varepsilon_2}(q+q')=\dfrac{1}{\varepsilon_1}q''\\ q-q'=q''\end{cases}\Rightarrow\begin{cases}q'=\dfrac{\varepsilon_2-\varepsilon_1}{\varepsilon_2+\varepsilon_1}q\\ q''=\dfrac{2\varepsilon_1}{\varepsilon_2+\varepsilon_1}q\end{cases}$

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分离变量法:

$\nabla^2\varphi = -\frac{\rho}{\varepsilon}$ 无源区域: $\nabla^2\varphi = 0$

$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0$

边界条件

适用条件. ① $\varphi(x,y,z) = X(x)\,Y(y)\,Z(z)$

② 区域边界与坐标面平行或重合.

$YZ\frac{\partial^2 X}{\partial x^2} + XZ\frac{\partial^2 Y}{\partial y^2} + XY\frac{\partial^2 Z}{\partial z^2} / XYZ = 0$

$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = 0$

令 $\frac{X''}{X} = \alpha^2$ $\frac{Y''}{Y} = \beta^2$ $\frac{Z''}{Z} = \gamma^2$ $\alpha^2+\beta^2+\gamma^2=0$

以 $\frac{X''}{X} = \alpha^2$ 为例:

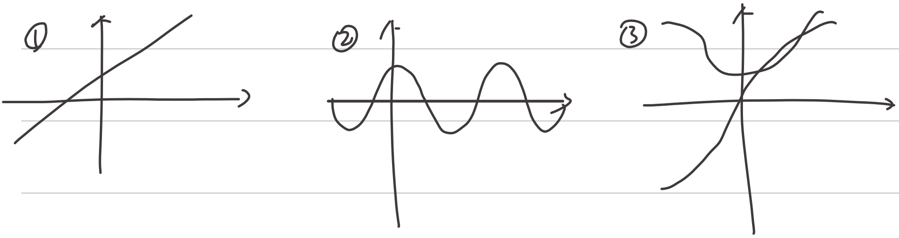
① 当 $\alpha^2 = 0$ 时. $X(x) = a_0 x + b_0$

② 当 $\alpha^2 < 0$ 时, 令 $\alpha = j\cdot k$, $X(x) = a_1\sin kx + a_2\cos kx$

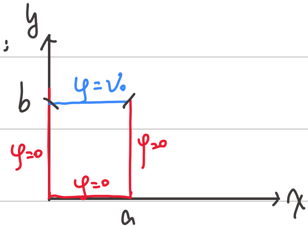
$= a_3\sin(kx+\theta)$ 周期性.

③ 当 $\alpha^2 > 0$ 时, 令 $\alpha^2 = kx$

$X(x) = b_1\,\mathrm{sh}\,kx + b_2\,\mathrm{ch}\,kx$ 非周期性.



例:



$\nabla^2\varphi=0$

① $\varphi(0,y)=0 \Rightarrow X(0)=0$

② $\varphi(a,y)=0 \Rightarrow X(a)=0$

③ $\varphi(x,0)=0 \Rightarrow Y(0)=0$

④ $\varphi(x,b)=V_0$

红线：导体槽 ：$\varphi=0$

蓝线：导体板

设 $\varphi(x,y)=X(x)\cdot Y(y)$

$$\frac{X''}{X}+\frac{Y''}{Y}=0$$

令 $\frac{X''}{X}=\alpha^2$. $\frac{Y''}{Y}=\beta^2 \Rightarrow \alpha^2+\beta^2=0$

$\alpha^2=0$. $X=a_0x+b_0$

$\alpha^2<0$ 令 $\alpha=jk$ $X=a_1\sin kx+a_2\cos kx$

$\alpha^2>0$ 令 $\alpha=k$ $y=b_1\,\mathrm{sh}\,kx+b_2\,\mathrm{ch}\,kx$

$\varphi(0,y)=X(0)Y(y)=0 \Rightarrow X(0)=0$

由①②至少有2个零点 $\Rightarrow \alpha^2<0$ 故 $\alpha^2<0$. 又 $\because \alpha^2+\beta^2=0$ $\therefore \beta^2>0$

$$\alpha=jk \quad \beta=k$$

$$\begin{cases} X(x)=a_1\sin kx+a_2\cos kx \\ Y(y)=b_1\,\mathrm{sh}\,ky+b_2\,\mathrm{ch}\,ky \end{cases}$$

$\mathrm{sh}=\frac{1}{2}(e^x-e^{-x})$

$\mathrm{ch}=\frac{1}{2}(e^x+e^{-x})$

由① $X(0)=a_2=0$.

② $X(a)=a_1\sin ak=0 \Rightarrow ak=n\pi \Rightarrow k=\frac{n\pi}{a}$

③ $Y(0)=b_2=0$

可得: $X(x)=a_1\sin\frac{n\pi}{a}x$ $Y(y)=b_1\,\mathrm{sh}\frac{n\pi}{a}y$

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① 通过一部分边界条件确定解的形式

线性、双曲函数只有一个零点

② 通过剩余边界条件确定线性叠加系数

$$\varphi_n = \sin\frac{n\pi x}{a}\,\mathrm{sh}\frac{n\pi y}{a}$$

通解: $\varphi(x,y) = \sum_n^{\infty} C_n \varphi_n$

$$\varphi(x,y) = \sum_{n=1}^{\infty} C_n \sin\frac{n\pi x}{a}\,\mathrm{sh}\frac{n\pi y}{a}$$

$\int_0^a \sin\frac{m\pi x}{a}dx = \frac{a}{m\pi}(1-\cos m\pi)$

$\int_0^a \sin\frac{n\pi x}{a}\sin\frac{m\pi x}{a}dx = \begin{cases} 0 & m\neq n \\ \frac{a}{2} & m=n \end{cases}$

由④ $\varphi(x,b) = \sum_{n=1}^{\infty} \underbrace{\boxed{C_n\,\mathrm{sh}\frac{n\pi b}{a}}}_{B_n} \sin\frac{n\pi x}{a} = V_0$

$$\sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{a} = V_0$$

$$\sum_{n=1}^{\infty} B_n \int_0^a \sin\frac{n\pi x}{a}\sin\frac{m\pi x}{a}dx = B_m\frac{a}{2}$$

$$\int_0^a V_0 \sin\frac{m\pi x}{a}dx = \frac{V_0 a}{m\pi}(1-\cos m\pi)$$

$\Rightarrow B_m\cdot\frac{\cancel{a}}{2} = \frac{V_0\cancel{a}}{m\pi}(1-\cos m\pi)$ 即 $B_n = \frac{2V_0}{n\pi}(1-\cos n\pi) = \begin{cases} \frac{4V_0}{n\pi} & n=1,3,5\cdots \\ 0 & n=2,4,6\cdots \end{cases}$

$$C_n = \frac{B_n}{\mathrm{sh}\frac{n\pi b}{a}} = \begin{cases} 0 & n=2,4\cdots \\ \frac{4V_0}{n\pi\,\mathrm{sh}\frac{n\pi b}{a}} & n=1,3,5\cdots \end{cases}$$

$$\varphi(x,y) = \sum_{n=1,3,5\cdots}^{a} \frac{4V_0}{n\pi\,\mathrm{sh}\frac{n\pi b}{a}} \sin\frac{n\pi x}{a}\,\mathrm{sh}\frac{n\pi y}{a}$$

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电磁能量的表现形式：
①储存在电磁场中. $W_e=\frac{1}{2}\vec{E}\cdot\vec{D}$. $W_m=\frac{1}{2}\vec{B}\cdot\vec{H}$.
②转换成热能(电热功率) $p=\vec{J}\cdot\vec{E}$
③在空间中流动,能流

① $\nabla\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t}$

② $\nabla\cdot(\vec{E}\times\vec{H})=\vec{H}\cdot(\nabla\times\vec{E})-\vec{E}\cdot(\nabla\times\vec{H})$

$$P=\int_V\vec{J}\cdot\vec{E}\,dV\overset{①}{=}\int_V\vec{E}\cdot(\nabla\times\vec{H}-\frac{\partial\vec{D}}{\partial t})dV$$

$\vec{E}\cdot(\nabla\times\vec{H})=\vec{H}\cdot(-\frac{\partial\vec{B}}{\partial t})-\nabla\cdot(\vec{E}\times\vec{H})$

$$\overset{②}{=}\int_V[-\nabla\cdot(\vec{E}\times\vec{H})-\vec{H}\cdot\frac{\partial\vec{B}}{\partial t}-\vec{E}\cdot\frac{\partial\vec{D}}{\partial t}]dV$$

③ $\frac{\partial}{\partial t}(\vec{A}\cdot\vec{B})=\frac{\partial\vec{A}}{\partial t}\cdot\vec{B}+\frac{\partial\vec{B}}{\partial t}\cdot\vec{A}$

若 $\vec{A}=\vec{B}$ 则

$\frac{1}{2}\cdot\frac{\partial}{\partial t}(\vec{A}\cdot\vec{A})=\vec{A}\cdot\frac{\partial\vec{A}}{\partial t}$

$$\Rightarrow\boxed{-\int_V\nabla\cdot(\vec{E}\times\vec{H})dV=\int_V(\vec{H}\cdot\frac{\partial\vec{B}}{\partial t}+\vec{E}\cdot\frac{\partial\vec{D}}{\partial t}+\vec{J}\cdot\vec{E})dV}$$

$\because$ $\vec{B}=\mu\vec{H}$ $\vec{D}=\varepsilon\vec{E}$ 则：

$\therefore$ $\frac{1}{2}\frac{\partial}{\partial t}(\vec{B}\cdot\vec{H})+\frac{1}{2}\frac{\partial}{\partial t}(\vec{D}\cdot\vec{E})$

即 $\frac{\partial}{\partial t}(\underbrace{\frac{1}{2}\vec{B}\cdot\vec{H}}_{W_m})+\frac{\partial}{\partial t}(\underbrace{\frac{1}{2}\vec{D}\cdot\vec{E}}_{W_e})$

坡印廷定理

即：$\boxed{-\int_V\nabla\cdot(\vec{E}\times\vec{H})dV=\frac{\partial}{\partial t}\int_V(W_m+W_e)dV+\int_V\vec{J}\cdot\vec{E}dV}$

从导体外流入的总功率=电磁场总能量的增加速率+电热功率

坡印廷矢量：$\vec{S}=\vec{E}\times\vec{H}$

$\int_V\nabla\cdot\vec{S}\,dV$：流出能量的功率

$\oint_S\vec{S}\cdot d\vec{S}$

$\vec{S}$ 从量纲上definitely流出功率的面密度.

$\nabla\cdot\vec{S}$：流出功率的体密度 → 真正能代表能量的流动

三种情况：①静电场.静磁场. $\vec{S}=\vec{E}\times\vec{H}$.

电荷不流动, $\vec{J}=0$, $\frac{\partial}{\partial t}(W_m+W_e)=0$ $\Rightarrow\int_V\nabla\cdot\vec{S}\,dV=0$ 流入的总功率为0

$\vec{S}$ 可能不为0 → 不代表能量功率流密度

$\nabla\cdot\vec{S}=0$ 真正有物理意义

② 恒定电流+静磁场 $\vec{J}=c$ $\frac{\partial}{\partial t}(W_m+W_e)=0$

$-\int_V\nabla\cdot\vec{S}\,dV=\int_V\vec{J}\cdot\vec{E}\,dV$ $\vec{S}$ 代表功率流密度.

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③时变电磁场：了解表瞬时功率能流密度.

波动方程:

均匀线性各向同性介质中，无源，无导电损耗，
$(\rho=0, \vec{J}=0)\quad(\sigma=0)$

$$\begin{cases}\nabla\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t}=\varepsilon\frac{\partial\vec{E}}{\partial t} & ①\\ \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}=-\mu\frac{\partial\vec{H}}{\partial t} & ②\\ \nabla\cdot\vec{H}=0 & ③\\ \nabla\cdot\vec{E}=0 & ④\end{cases}$$

对②取旋度: $\nabla\times(\nabla\times\vec{E})=\nabla(\nabla\cdot\vec{E})-\nabla^2\vec{E}=\boxed{-\nabla^2\vec{E}}$

$$=-\mu\left(\nabla\times\frac{\partial\vec{H}}{\partial t}\right)=-\mu\frac{\partial}{\partial t}(\nabla\times\vec{H})$$

$$=\boxed{-\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}}$$

$$\Rightarrow\begin{cases}\nabla^2\vec{E}-\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}=0\\ \nabla^2\vec{H}-\mu\varepsilon\frac{\partial^2\vec{H}}{\partial t^2}=0\end{cases}$$

波动方程

矢量亥姆霍兹方程

复数形式:

$$\nabla^2\dot{\vec{E}}-\mu\varepsilon(j\omega)^2\dot{\vec{E}}=0$$

$$\nabla^2\dot{\vec{E}}+\mu\varepsilon\omega^2\dot{\vec{E}}=0$$

$k=\omega\sqrt{\mu\varepsilon}$

$$\begin{cases}\nabla^2\dot{\vec{E}}+k^2\dot{\vec{E}}=0\\ \nabla^2\dot{\vec{H}}+k^2\dot{\vec{H}}=0\end{cases}$$

相速: $v=\frac{1}{\sqrt{\varepsilon\mu}}$

$$c=\frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

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辅助位函数：　　$\nabla\cdot\vec{B}=0$　　$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}=-\frac{\partial}{\partial t}(\nabla\times\vec{A})$

$=\nabla\times(-\frac{\partial\vec{A}}{\partial t})$

矢量位 $\vec{A}$: $\vec{B}=\nabla\times\vec{A}$　矢量磁位　$\vec{B}=\nabla\times\vec{A}$

$\nabla\times(\underline{\vec{E}+\frac{\partial\vec{A}}{\partial t}})=0$

标量位 φ: $\vec{E}+\frac{\partial\vec{A}}{\partial t}=-\nabla\varphi$　　$\vec{E}+\frac{\partial\vec{A}}{\partial t}=-\nabla\varphi$

位函数不唯一，要确定位函数，需要规定 $\boxed{\nabla\cdot\vec{A}=0}$

$\nabla\cdot\vec{E}=\frac{\rho}{\varepsilon}\Rightarrow\nabla\cdot(-\nabla\varphi-\frac{\partial\vec{A}}{\partial t})=\frac{\rho}{\varepsilon}$　　$\nabla^2\varphi+\frac{\partial}{\partial t}(\nabla\cdot\vec{A})=-\frac{\rho}{\varepsilon}$

$\nabla\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t}\Rightarrow\frac{1}{\mu}\nabla\times(\nabla\times\vec{A})=\vec{J}+\frac{\partial\vec{D}}{\partial t}$

左边 $=\frac{1}{\mu}[\nabla(\nabla\cdot\vec{A})-\nabla^2\vec{A}]$

右边 $=\vec{J}+\varepsilon\frac{\partial\vec{E}}{\partial t}=\vec{J}+\varepsilon\frac{\partial\vec{E}}{\partial t}=\vec{J}+\varepsilon\frac{\partial}{\partial t}(-\nabla\varphi-\frac{\partial\vec{A}}{\partial t})$

整理可得: $\nabla^2\vec{A}-\mu\varepsilon\frac{\partial^2\vec{A}}{\partial t^2}=-\mu\vec{J}+\nabla(\nabla\cdot\vec{A}+\mu\varepsilon\frac{\partial\varphi}{\partial t})$

洛伦兹规范: $\nabla\cdot\vec{A}=-\mu\varepsilon\frac{\partial\varphi}{\partial t}$

$$\begin{cases}\nabla^2\varphi-\mu\varepsilon\frac{\partial^2\varphi}{\partial t^2}=-\frac{\rho}{\varepsilon}\\ \nabla^2\vec{A}-\mu\varepsilon\frac{\partial^2\vec{A}}{\partial t^2}=-\mu\vec{J}\end{cases}$$

位函数的场方程
达朗贝尔方程

$\nabla\cdot\vec{B}=0$　　$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}=-\frac{\partial(\nabla\times\vec{A})}{\partial t}$

$\underline{\vec{B}=\nabla\times\vec{A}}$　　$\nabla\times\vec{E}=\nabla\times(-\frac{\partial\vec{A}}{\partial t})$

$\nabla\times(\vec{E}+\frac{\partial\vec{A}}{\partial t})=0$

$\vec{E}-\frac{\partial\vec{A}}{\partial t}=-\nabla\varphi$

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正弦电磁场

$$\vec{E}(x,y,z,t) = E_x(x,y,z,t)\vec{e}_x + E_y(x,y,z,t)\vec{e}_y + E_z(x,y,z,t)\vec{e}_z$$

$E_x(x,y,z,t) = \underbrace{E_{xm}(x,y,z)}_{\text{振幅}} \cos[\underbrace{\omega}_{\text{角频率}} t + \underbrace{\varphi_x(x,y,z)}_{\text{初相位}}]$ 　　E_y. E_z 同理

$= Re[E_{xm} e^{j(\omega t+\varphi_x)}]$

$= Re[\underbrace{E_{xm} e^{j\varphi_x}}_{\dot{E}_{xm}\text{(复振幅)}} \cdot e^{j\omega t}] = Re[\dot{E}_{xm} e^{j\omega t}]$ 　复数表示

$E_x \longleftrightarrow \dot{E}_{xm}$

$\vec{E}(x,y,z,t) = Re[\underbrace{(\dot{E}_{xm}\vec{e}_x + \dot{E}_{ym}\vec{e}_y + \dot{E}_{zm}\vec{e}_z)}_{\text{总复振幅}} e^{j\omega t}]$

$\frac{\partial E_x}{\partial t} \longleftrightarrow j\omega \dot{E}_{xm}$

$$\frac{\partial E_x(x,y,z,t)}{\partial t} = -E_{xm}\omega\sin(\omega t+\varphi_x) = Re(\underline{j\omega \dot{E}_{xm}} e^{j\omega t})$$

$\frac{\partial E_x}{\partial t} \longleftrightarrow j\omega \dot{E}_{xm}$

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场方程的复数形式:

$$\begin{cases}\nabla\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t} & \Rightarrow \nabla\times\dot{\vec{H}}=\dot{\vec{J}}+j\omega\dot{\vec{D}}\\ \nabla\cdot\vec{B}=0 & \Rightarrow \nabla\times\dot{\vec{B}}=0\\ \nabla\cdot\vec{D}=\rho & \Rightarrow \nabla\cdot\dot{\vec{D}}=\dot{\rho}\\ \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} & \Rightarrow \nabla\times\dot{\vec{E}}=-j\omega\dot{\vec{B}}\end{cases}$$

$$\vec{H}=Re[\dot{\vec{H}}\cdot e^{j\omega t}] \qquad \vec{J}=Re(\dot{\vec{J}}\cdot e^{j\omega t}) \qquad \frac{\partial\vec{D}}{\partial t}=j\omega\cdot\dot{\vec{D}}\cdot e^{j\omega t}$$

$$\nabla\times Re(\dot{\vec{H}}\cdot e^{j\omega t})=Re(\dot{\vec{J}}e^{j\omega t})+Re(j\omega\dot{\vec{D}}e^{j\omega t})$$

$$Re(\nabla\times\dot{\vec{H}}e^{j\omega t})=Re[(\dot{\vec{J}}+j\omega\dot{\vec{D}})\cdot e^{j\omega t}]$$

$$Re[(\nabla\times\dot{\vec{H}}-\dot{\vec{J}}-j\omega\dot{\vec{D}})e^{j\omega t}]=0 \Rightarrow \nabla\times\dot{\vec{H}}=\dot{\vec{J}}+j\omega\dot{\vec{D}}$$

复坡印廷矢量:

$$\vec{S}(\vec{r},t)=\vec{E}(\vec{r},t)\times\vec{H}(\vec{r},t) \qquad\qquad Re(a)=\frac{1}{2}(a+a^*)$$

$$=Re[\dot{\vec{E}}(\vec{r})e^{j\omega t}]\times Re[\dot{\vec{H}}(\vec{r})e^{j\omega t}]$$

$$=\frac{1}{2}[\dot{\vec{E}}(\vec{r})e^{j\omega t}+\dot{\vec{E}}^*(\vec{r})e^{-j\omega t}]\times\frac{1}{2}[\dot{\vec{H}}(\vec{r})e^{j\omega t}+\dot{\vec{H}}^*(\vec{r})e^{-j\omega t}]$$

$$=\frac{1}{4}[\dot{\vec{E}}(\vec{r})\times\dot{\vec{H}}(\vec{r})e^{j2\omega t}+\dot{\vec{E}}^*(\vec{r})\times\dot{\vec{H}}^*(\vec{r})e^{-j2\omega t}+\dot{\vec{E}}(\vec{r})\times\dot{\vec{H}}^*(\vec{r})+\dot{\vec{E}}^*(\vec{r})\times\dot{\vec{H}}(\vec{r})]$$

$$=\frac{1}{2}Re[\dot{\vec{E}}(\vec{r})\times\dot{\vec{H}}(\vec{r})\boxed{e^{j2\omega t}}]+\frac{1}{2}Re[\dot{\vec{E}}(\vec{r})\times\dot{\vec{H}}^*(\vec{r})]$$

$$\boxed{\begin{aligned}&\vec{S}_{av}=\frac{1}{T}\int_0^T\vec{S}\,dt=Re[\frac{1}{2}\dot{\vec{E}}(\vec{r})\times\dot{\vec{H}}^*(\vec{r})]\\&\vec{S}(\vec{r})=\frac{1}{2}\dot{\vec{E}}(\vec{r})\times\dot{\vec{H}}^*(\vec{r}) \quad \text{复坡印廷矢量}\end{aligned}}$$

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电场能量密度 $w_e(\vec r,t)=\frac{1}{2}\vec E(\vec r,t)\cdot\vec D(\vec r,t)$

$=\frac{1}{4}Re[\dot{\vec E}(\vec r)\cdot\vec D(\vec r)e^{j2\omega t}]+\frac{1}{4}Re[\vec E(\vec r)\cdot\vec D^*(\vec r)]$

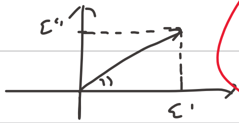
平均: $W_{ave}=\frac{1}{4}Re[\vec E(\vec r)\cdot\vec D^*(\vec r)]$

磁能密度: $W_m=\frac{1}{2}\vec B(\vec r,t)\cdot\vec H(\vec r,t)$

平均: $W_{avm}=\frac{1}{4}Re[\vec B(\vec r)\cdot\vec H^*(\vec r)]$

电热功率(损耗)密度: $p(\vec r,t)=\vec J(\vec r,t)\cdot\vec E(\vec r,t)$

平均: $p_{av}=\frac{1}{2}Re[\vec J(\vec r)\cdot\vec E^*(\vec r)]$



损耗角 δ_ε

$\tan\delta_\varepsilon=\frac{\varepsilon''}{\varepsilon'}$

复介电常数和复磁导率

$\varepsilon_c=\varepsilon'(\omega)-j\varepsilon''(\omega)$ ε'反映了储能能力. ε''反映了介质的极化损耗.

$\mu_c=\mu'(\omega)-j\mu''(\omega)$ μ'反映了储能能力, μ''反映了磁化损耗

$p=\frac{1}{2}Re[\vec J\cdot\vec E^*]$ $\vec J=\underbrace{\vec J_c+\vec J_v}_{=0}+\vec J_d=\frac{\partial\vec D}{\partial t}$

$=\frac{1}{2}Re[j\omega\vec D\cdot\vec E^*]$ $\vec J_d=\frac{\partial\vec D}{\partial t}$ $\vec J=j\omega\vec D$

损耗角 δ_μ

$\tan\delta_\mu=\frac{\mu''}{\mu'}$

$=\frac{1}{2}Re[j\omega\varepsilon_c\vec E\cdot\vec E^*]$ $\vec D=\varepsilon_c\vec E$

$=\frac{1}{2}Re[j\omega(\varepsilon'-j\varepsilon'')\cdot E_m^2]$ 等效复介电常数 ε_c'

$=\frac{1}{2}Re[j\omega\varepsilon' E_m^2+\omega\varepsilon'' E_m^2]$ $\nabla\times\vec H=\vec J+j\omega\vec D$

$\boxed{=\frac{1}{2}\omega\varepsilon'' E_m^2}$

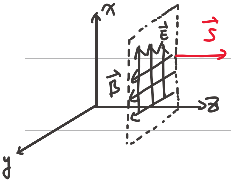
$=\sigma\vec E+j\omega(\varepsilon_c\vec E)=j\omega[\underbrace{\varepsilon'-j(\varepsilon''+\frac{\sigma}{\omega})}_{\varepsilon_c'}]\vec E$

$=j\omega\varepsilon_c'\vec E$ 等效位移电流

均匀平面电磁波：

等相位面：
1. 球状波
2. 柱状波
3. 平面波

均匀平面波：等相位面上各点的场量相同



$\vec{S}$ 若取合适的坐标系

$\vec{E} = E_x(z,t)\,\vec{e}_x$

$\vec{H} = H_y(z,t)\,\vec{e}_y$

坡印廷矢量：$\vec{S} = \vec{E}\times\vec{H}$

无源、无损耗区域：

$$\begin{cases} \nabla^2 \vec{E} - \mu\varepsilon \dfrac{\partial^2 \vec{E}}{\partial t^2} = 0 \\ \nabla^2 \vec{H} - \mu\varepsilon \dfrac{\partial^2 \vec{H}}{\partial t^2} = 0 \end{cases}$$

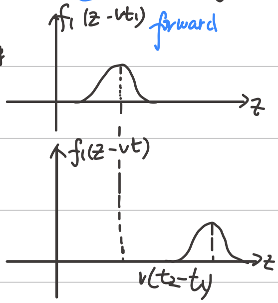
$$\nabla^2 E_x - \mu\varepsilon \frac{\partial^2 E_x}{\partial t^2} = 0 \qquad \nabla^2 E_x = \underbrace{\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2}}_{=0} + \frac{\partial^2 E_x}{\partial z^2}$$

$$\begin{cases} \dfrac{\partial^2 E_x}{\partial z^2} - \mu\varepsilon \dfrac{\partial^2 E_x}{\partial t^2} = 0 \\ \dfrac{\partial^2 H_y}{\partial z^2} - \mu\varepsilon \dfrac{\partial^2 H_y}{\partial t^2} = 0 \end{cases}$$

通解：$E_x(z,t) = \underbrace{f_1(z-vt)}_{\text{forward}} + \underbrace{f_2(z+vt)}_{\text{backward}}$ 其中 f_1, f_2 为任意函数

$t = t_1$ 时

$t = t_2$ 时



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正弦均匀平面波：无源、无损耗区域

$$\nabla^2\dot{\vec{E}} + k^2\dot{\vec{E}} = 0 \Rightarrow \frac{d^2\dot{E}_x(z)}{dz^2} + k^2\dot{E}_x(z) = 0$$

通解：$\dot{E}_x(z) = E_0^+ e^{-jkz} + E_0^- e^{jkz}$

$$\dot{\vec{E}} = \dot{E}_x(z)\vec{e}_x = (E_0^+ e^{-jkz} + E_0^- e^{jkz})\vec{e}_x.$$

$$\nabla\times\dot{\vec{E}} = -j\omega\mu\dot{\vec{H}} \Rightarrow \dot{\vec{H}} = (H_0^+ e^{-jkz} + H_0^- e^{jkz})\vec{e}_y$$

$\eta = \frac{E_0^+}{H_0^+} = -\frac{E_0^-}{H_0^-} = \sqrt{\frac{\mu}{\varepsilon}}$ 波阻抗（单位：Ω）

真空中 $\eta = 120\pi\,\Omega$

传播特性：

$\dot{\vec{E}} = E_0 e^{-jkz}\vec{e}_x$　　$E_{0m} = |\dot{E}_0|$

$\vec{E} = \mathrm{Re}(\dot{\vec{E}} e^{j\omega t}) = \vec{e}_x E_{0m}\cos(\omega t - kz + \varphi_0)$

$\vec{H} = \vec{e}_y H_{0m}\cos(\underbrace{\omega t}_{\text{时间相位}} - \underbrace{kz}_{\text{空间相位}} + \underbrace{\varphi_0}_{\text{初相位}})$

等相面方程 $\omega t - kz + \varphi_0 = c$

$k = \omega\sqrt{\mu\varepsilon}$

相速：等相面传播的速度 $\boxed{v_p = \frac{dz}{dt} = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}}}$

波长：空间相位变化 2π 所经过的距离 $k\lambda = 2\pi \Rightarrow \lambda = \frac{2\pi}{k}$

周期：时间相位变化 2π - - - -　$\omega \cdot T = 2\pi \Rightarrow T = \frac{2\pi}{\omega}$

频率：$f = \frac{1}{T} = \frac{\omega}{2\pi}$　　$\boxed{v_p = f\cdot\lambda = \frac{\lambda}{T}}$

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复坡印廷矢量:

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \vec{e}_z\frac{E_{0m}^2}{2\eta}$$

$$\vec{S}_{av} = Re[\vec{S}] = \vec{e}_z\cdot\frac{E_{0m}^2}{2\eta}$$

电场能密度瞬时值

$$W_e(t) = \frac{1}{2}\vec{D}\cdot\vec{E} = \frac{1}{2}\varepsilon E_{0m}^2\cos^2(wt-kz+\phi_0)$$

平均值: $W_{ave}(t) = \frac{1}{4}\varepsilon E_{0m}^2$

磁能密度瞬时值.

$$W_m(t) = \frac{1}{2}\vec{H}\cdot\vec{B} = \frac{1}{2}\mu H_{0m}^2\cos^2(wt-kz+\phi_0) = W_e(t)$$

平均值: $W_{av,m} = \frac{1}{4}\mu H_{0m}^2 = W_{av,e}$

能速: $\vec{v}_e = \frac{\vec{S}_{av}}{W_{av}} = \vec{e}_z\frac{1}{\sqrt{\mu\varepsilon}}$

导电媒质中的均匀平面波

复介电常数 $\varepsilon_c = \varepsilon - j\frac{\sigma}{w}$

$$\begin{cases}\nabla\times\vec{H} = \sigma\vec{E} + jw\varepsilon\vec{E} = jw\varepsilon_c\vec{E}\\ \nabla\times\vec{E} = -jw\mu\vec{H}\\ \nabla\cdot\vec{H} = 0\\ \nabla\cdot\vec{E} = 0\end{cases}$$

$$\Rightarrow\begin{cases}\nabla^2\vec{E} + \gamma^2\vec{E} = 0\\ \nabla^2\vec{H} + \gamma^2\vec{H} = 0\\ \gamma = w\sqrt{\mu\varepsilon_c}\end{cases}$$

$$\vec{E} = \vec{e}_x E_0 e^{-j\gamma z} = \vec{e}_x E_{0m} e^{j\phi_0} e^{-j\gamma z}$$

$\gamma = \beta - \alpha j$

$$= \vec{e}_x E_{0m} e^{j\phi_0} e^{-\alpha z}\cdot e^{-j\beta z}$$

$$\boxed{\text{瞬时值}: \vec{E} = Re(\vec{E}e^{jwt}) = \vec{e}_x E_{0m}\boxed{e^{-\alpha z}}\cos(wt-\beta z+\phi_0)}$$

衰减因子

α: 衰减常数.

β: 相位常数

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$$\begin{cases}\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]} \\ \beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}\end{cases}$$

$$\vec{H} = \frac{\nabla\times\vec{E}}{-j\omega\mu} = \vec{e}_y\frac{E_0}{\eta_c}e^{-j\gamma z} \qquad 令 H_0 = \frac{E_0}{\eta_c} \ (复数)$$

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon - j\frac{\sigma}{\omega}}}$$ 波阻抗

$$设 \eta_c = |\eta_c|e^{j\theta} \qquad \vec{H} = \vec{e}_y\frac{E_0}{|\eta_c|}e^{-j\theta}\cdot e^{-j\gamma z} \qquad \gamma = \beta - \alpha j$$

$$= \vec{e}_y\frac{E_0}{|\eta_c|}e^{-\alpha z}e^{-j(\beta z+\theta)}$$

$$\vec{H} = Re(\dot{\vec{H}}\cdot e^{j\omega t})$$

$$\underline{\vec{H}(z,t) = \vec{e}_y\frac{E_{0m}}{|\eta_c|}e^{-\alpha z}\cos(\omega t - \beta z + \phi_0 - \theta)}$$ 与$\vec{E}$之间有相位差.

<u>电场超前磁场</u>

坡印廷矢量: $\dot{\vec{S}} = \frac{1}{2}\dot{\vec{E}}\times\dot{\vec{H}}^*$ $\qquad S_{av} = \frac{1}{T}\int_0^T \vec{S}\cdot dt$

$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}$ $\qquad w_{av,e} = \frac{1}{4}\varepsilon|\dot{\vec{E}}|^2$

$\qquad w_{av,m} = \frac{1}{4}\varepsilon|\dot{\vec{H}}|^2$

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良导体中的均匀平面波

当 $\frac{\sigma}{\omega\varepsilon} < 0.01$，电介质

$0.01 < \frac{\sigma}{\omega\varepsilon} < 100$，半导体或不良导体

$\frac{\sigma}{\omega\varepsilon} \geq 100$，良导体

$\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega}$

传导电流

损耗角：$\tan\delta_e = \frac{\sigma}{\omega\varepsilon} = \left|\frac{\vec{J}_c}{\vec{J}_d}\right|$

位移电流

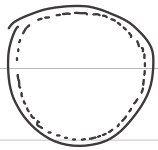
$\vec{E} = \vec{e}_x E_0 e^{-\alpha z} e^{-j\beta z}$

$$\begin{cases} \alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} \approx \sqrt{\frac{\omega\mu\sigma}{2}} \\ \beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]} \approx \sqrt{\frac{\omega\mu\sigma}{2}} \end{cases}$$

$v_p = \sqrt{\frac{2\omega}{\mu\sigma}}$　$\lambda = 2\pi\sqrt{\frac{2}{\omega\mu\sigma}}$

$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\, e^{j\frac{\pi}{4}}$　波阻抗

集肤效应



集肤深度

GHz　μm

E的振幅衰减到表面处的 $\frac{1}{e}$

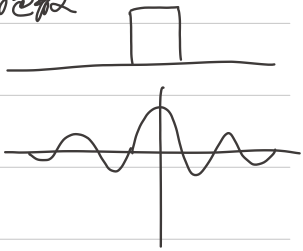
$e^{-\alpha z} = \frac{1}{e} \Rightarrow \boxed{z = \frac{1}{\alpha}} = \sqrt{\frac{2}{\omega\mu\sigma}}$

日期： /

色散与群速

相速 $V_p=\frac{\omega}{\beta}$ $V_p(\omega)$ V_p与ω相关 ⇒ 波的色散

单色波、调制波



群速 V_g：包络波的推进速度（波群速度）

$$\vec{E}=\vec{E_1}+\vec{E_2}$$

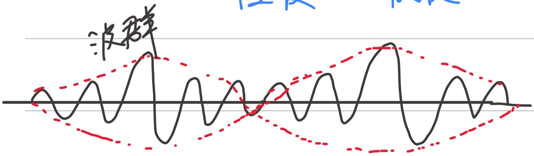
$$\vec{E_1}=E_0\cos[(\omega+\Delta\omega)t-(\beta+\Delta\beta)z]$$
$$\vec{E_2}=E_0\cos[(\omega-\Delta\omega)t-(\beta-\Delta\beta)z]$$

$\begin{cases}\Delta\omega\ll\omega\\ \Delta\beta\ll\beta\end{cases}$

振幅，包络波

$$\vec{E}=\boxed{2E_0\cos(\Delta\omega t-\Delta\beta z)}\cos(\omega t-\beta z)$$

慢波 快波



窄带波

$$V_g=\frac{\Delta\omega}{\Delta\beta}=\frac{d\omega}{d\beta}$$

相速与群速的关系：$V_g=\frac{d\omega}{d\beta}=\frac{d(V_p\beta)}{d\beta}=V_p+\beta\frac{dV_p}{d\beta}=V_p+\frac{\omega}{V_p}\frac{dV_p}{d\omega}\cdot V_g$

$$\frac{\beta}{d\beta}=\frac{1}{\frac{1}{\beta}}\cdot\frac{1}{d\beta}=\frac{\frac{\omega}{\omega}}{\beta}\frac{d\omega}{d\omega d\beta}$$
$$=\frac{\omega}{V_p}\cdot\frac{V_g}{d\omega}$$

$$V_g=\frac{V_p}{1-\frac{\omega}{V_p}\frac{dV_p}{d\omega}}$$

$\frac{dV_p}{d\omega}=0$ $V_g=V_p$ 无色散频

<0 $V_g<V_p$ 正常色散频

>0 $V_g>V_p$ 反常色散频

（相速有可能大于光速）

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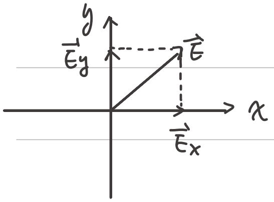
电磁波的极化:

电场强度$\vec{E}$的空间取向随时间变化的方式,叫作极化方式

线极化,圆极化,椭圆极化



$$\vec{E}_x = \vec{e}_x E_{xm} \cos(\omega t + \phi_x)$$

$$\vec{E}_y = \vec{e}_y E_{ym} \cos(\omega t + \phi_y)$$

$$\vec{E} = \vec{E}_x + \vec{E}_y$$

① 线极化: $|\phi_x - \phi_y| = 0$ 或 π　　以$\phi_x = \phi_y$为例

$$\vec{E} = \vec{E}_x + \vec{E}_y$$

$$= \vec{e}_x E_{xm} \cos(\omega t + \phi_x) + \vec{e}_y E_{ym} \cos(\omega t + \phi_y)$$

$$= (\vec{e}_x E_{xm} + \vec{e}_y E_{ym}) \cos(\omega t + \phi_x)$$

② 圆极化: 当 $E_{xm} = E_{ym} = E_m$, $|\phi_x - \phi_y| = \frac{\pi}{2}$ 或 $\frac{3}{2}\pi$ 时　　以$|\phi_x - \phi_y| = \frac{\pi}{2}$为例

$$\vec{E} = \vec{e}_x E_m \cos(\omega t + \phi_x) + \vec{e}_y E_m \cos(\omega t + \phi_x \mp \frac{\pi}{2})$$

$$= \vec{e}_x E_m \cos(\omega t + \phi_x) \pm \vec{e}_y E_m \sin(\omega t + \phi_x)$$

$$= E_m [\vec{e}_x \cos(\omega t + \phi_x) \pm \vec{e}_y \sin(\omega t + \phi_x)]$$

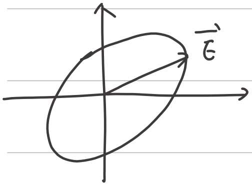
$|\vec{E}| = E_m$　　$\tan\alpha = \frac{|E_y|}{|E_x|} = \pm \tan(\omega t + \phi_x)$

$\alpha = \omega t + \phi_x$ 逆时针 右旋　　$\alpha = -(\omega t + \phi_x)$ 顺时针 左旋

日期:　　/

③ 椭圆极化：第一般情况

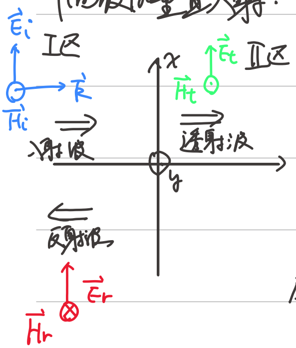
$$\frac{|\vec{E_x}|^2}{E_{xm}^2} - 2\frac{|\vec{E_x}|}{E_{xm}}\frac{|\vec{E_y}|}{E_{ym}}\cos(\phi_x-\phi_y) + \frac{|\vec{E_y}|^2}{E_{ym}^2} = \sin^2(\phi_x-\phi_y)$$



$0 < \phi_x - \phi_y < \pi$　右旋极化

$-\pi < \phi_x - \phi_y < 0$　左旋极化

平面波的垂直入射.



I区: 无耗介质(真空)　　II区: 理想导体

入射波 $\left\{ \begin{array}{l} \vec{E}_i = \vec{e}_x E_{i0} e^{-jk_1 z}, \quad k_1 = \omega\sqrt{\mu_1 \varepsilon_1} \\ \vec{H}_i = \vec{e}_y \dfrac{E_{i0}}{\eta_1} e^{-jk_1 z}, \quad \eta_1 = \sqrt{\dfrac{\mu_1}{\varepsilon_1}} \end{array} \right.$

反射波: $\left| \begin{array}{l} \vec{E}_r = \vec{e}_x E_{r0} e^{jk_1 z} \\ \vec{H}_r = -\vec{e}_y \dfrac{E_{r0}}{\eta_1} e^{jk_1 z}. \end{array} \right.$

透射波: $\vec{E}_t = 0, \quad \vec{H}_t = 0$

由边界条件: $\vec{E}_1 = \vec{E}_i + \vec{E}_r = \vec{E}_2 = 0 \quad (z=0)$

$$\vec{e}_x(E_{i0} + E_{r0}) = 0 \Rightarrow \boxed{E_{r0} = -E_{i0}}$$

$$\boxed{\text{反射系数 } \Gamma = \frac{E_{r0}}{E_{i0}} = -1}$$

$$\vec{E}_1 = \vec{E}_i + \vec{E}_r = \vec{e}_x E_{i0}(e^{-jk_1 z} - e^{jk_1 z}) = \vec{e}_x 2j E_{i0} \sin(k_1 z)$$

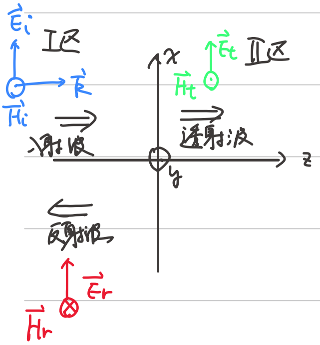
$$\vec{H}_1 = \vec{H}_i + \vec{H}_r = \vec{e}_y \frac{E_{i0}}{\eta_1}(e^{-jk_1 z} + e^{jk_1 z}) = \vec{e}_y \frac{2E_{i0}}{\eta_1} \cos(k_1 z)$$

$$\boxed{\begin{array}{l} \vec{E}_1 = \mathrm{Re}(\vec{E}_1 e^{j\omega t}) = \vec{e}_x 2E_{i0} \sin(k_1 z) \sin\omega t \\ \vec{H}_1 = \mathrm{Re}(\vec{H}_1 \cdot e^{j\omega t}) = \vec{e}_y 2 \dfrac{E_{i0}}{\eta_1} \cos(k_1 z) \cos\omega t \end{array}}$$

振幅 $= 2E_{i0} \sin k_1 z$　　当 $z = n\pi$ 时 振幅为0　　波节面

当 $z = n\pi + \frac{\pi}{2}$ 时, 振幅 $= 2E_{i0}$　　波腹面

驻波. 不传递能量　$\vec{S}_{av} = \mathrm{Re}[\vec{S} \cdot e^{j\omega t}] = 0$



I区：理想介质　　II区：理想介质

$$\vec{E}_i = \vec{e}_x E_{i0} e^{-jk_1 z},\quad k_1=\omega\sqrt{\mu_1\varepsilon_1}$$

入射波 $\left\{ \vec{H}_i = \vec{e}_y \frac{E_{i0}}{\eta_1} e^{-jk_1 z},\quad \eta_1=\sqrt{\frac{\mu_1}{\varepsilon_1}} \right.$

反射波：
$$\vec{E}_r = \vec{e}_x E_{r0} e^{jk_1 z}$$
$$\vec{H}_r = -\vec{e}_y \frac{E_{r0}}{\eta_1} e^{jk_1 z}.$$

边界条件：

由z=0两侧 电场切向分量连续

$$E_{i0}+E_{r0}=E_{t0}$$

由z=0两侧，切向磁场连续

$$\frac{E_{i0}}{\eta_1}-\frac{E_{r0}}{\eta_1}=\frac{E_{t0}}{\eta_2}$$

$$\begin{cases} E_{r0}=\dfrac{\eta_2-\eta_1}{\eta_1+\eta_2}E_{i0} \\ E_{t0}=\dfrac{2\eta_2}{\eta_1+\eta_2}E_{i0} \end{cases}$$

$\vec{n}\times(\vec{H}_1-\vec{H}_2)=\vec{J}_s$.

$H_{1t}-H_{2t}=|J_s|$

$=0$（理想介质无电流）

反射系数 $\Gamma=\dfrac{E_{r0}}{E_{i0}}=\dfrac{\eta_2-\eta_1}{\eta_2+\eta_1}$

透射系数 $T=\dfrac{E_{t0}}{E_{i0}}=\dfrac{2\eta_2}{\eta_2+\eta_1}$

$\boxed{1+\Gamma=T}$ 恒成立.

I区：
$$\vec{E}_1=\vec{E}_i+\vec{E}_r=\vec{e}_x E_{i0}(e^{-jk_1z}+\Gamma e^{jk_1z})$$
$$=\vec{e}_x E_{i0}[(e^{-jk_1z}+\Gamma e^{-jk_1z})+(\Gamma e^{jk_1z}-\Gamma e^{-jk_1z})]$$
$$=\vec{e}_x E_{i0}[\underbrace{Te^{-jk_1z}}_{行波}+\underbrace{\Gamma 2j\sin(k_1z)}_{驻波}]$$ 行驻波

当 $k_1z=n\pi$ 时，振幅最大 $E_{max}=(1+|\Gamma|)E_{i0}$

当 $k_1z=\frac{\pi}{2}+n\pi$，振幅最小 $E_{min}=(1-|\Gamma|)E_{i0}$

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$$\boxed{\text{驻波比}: S = \frac{E_{max}}{E_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}}$$ 当 $S=1$ 时. $\Gamma=0$ 无驻波

$$\vec{H}_1 = \vec{H}_i + \vec{H}_r = \vec{e}_y \frac{E_{i0}}{\eta_1}\left[(1+\Gamma)e^{-jk_1z} - 2\Gamma\cos k_1 z\right]$$

II区:

$$\begin{cases} \vec{E}_2 = \vec{e}_x E_{i0} \dfrac{2\eta_2}{\eta_1+\eta_2} e^{-jk_2z} \\ \vec{H}_2 = \vec{e}_y E_{i0} \dfrac{2}{\eta_1+\eta_2} e^{-jk_2z} \end{cases}$$ 行波. 无驻波

$$\vec{S}_{av} = Re(\vec{S}) = Re\left(\frac{1}{2}\vec{E}\times\vec{H}^*\right)$$

$$\vec{S}_{av,i} = \vec{e}_z \cdot \frac{1}{2}\frac{1}{\eta_1} E_{i0}^2$$

$$\vec{S}_{av,r} = -\vec{e}_z \cdot \frac{|\Gamma|^2}{2\eta_1} E_{i0}^2 = -|\Gamma|^2 \vec{S}_{av,i}$$

$$\vec{S}_{av,t} = \vec{e}_z \frac{|T|^2}{2\eta_2} E_{i0}^2 = \frac{\eta_1}{\eta_2}|T|^2 \vec{S}_{av,i}$$

$$\boxed{\vec{S}_{av,i} + \vec{S}_{av,r} = \vec{S}_{av,t}}$$ 能量守恒

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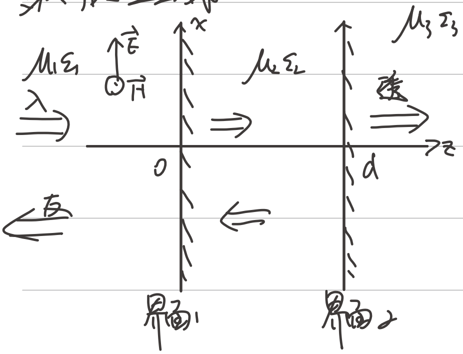
多层介质垂直入射



为反射波与入射波的比值

$$\begin{cases} \vec{E}_1 = \vec{e}_x E_{i0}^1 \left(e^{-jk_1 z} + \underline{\Gamma_1} e^{jk_1 z}\right) \\ \vec{H}_1 = \vec{e}_y \dfrac{E_{i0}^1}{\eta_1} \left(e^{-jk_1 z} - \Gamma_1 e^{jk_1 z}\right) \end{cases}$$

$$\begin{cases} \vec{E}_2 = \vec{e}_x E_{i0}^2 \left[e^{-jk_2(z-d)} + \Gamma_2 e^{jk_2(z-d)}\right] \\ \vec{H}_2 = \vec{e}_y \dfrac{E_{i0}^2}{\eta_2} \left[e^{-jk_2(z-d)} - \Gamma_2 e^{jk_2(z-d)}\right] \end{cases}$$

$$\boxed{\Gamma_2 = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2}}$$

由z=0处的边界条件

$$\vec{E}_1(z=0) = \vec{E}_2(z=0) \Rightarrow E_{i0}^1 (1+\Gamma_1) = E_{i0}^2 \left(e^{jk_2 d} + \Gamma_2 e^{-jk_2 d}\right)$$

$$\vec{H}_1(z=0) = \vec{H}_2(z=0) \Rightarrow \frac{E_{i0}^1}{\eta_1}(1-\Gamma_1) = \frac{E_{i0}^2}{\eta_2}\left(e^{jk_2 d} - \Gamma_2 e^{-jk_2 d}\right)$$

$$\Rightarrow \eta_1 \frac{1+\Gamma_1}{1-\Gamma_1} = \boxed{\eta_2 \frac{e^{jk_2 d} + \Gamma_2 e^{-jk_2 d}}{e^{jk_2 d} - \Gamma_2 e^{-jk_2 d}}} = Z_p$$

等效阻抗

多层介质：$\Gamma_1 = \dfrac{Z_p - \eta_1}{Z_p + \eta_1}$

两层介质：$\Gamma_1 = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

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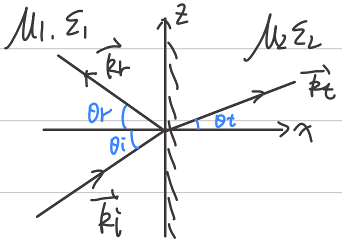
斜入射



$$\begin{cases} \theta_i = \theta_r \\ \dfrac{\sin\theta_t}{\sin\theta_i} = \dfrac{n_2}{n_1} \end{cases}$$

$$\vec{r} = x\vec{e_x} + y\vec{e_y} + z\vec{e_z}$$

$$\vec{k} = |k|\vec{e_k} \qquad |k| = w\sqrt{\mu\varepsilon} \qquad |\vec{E}| = E_{om} e^{j(wt - \vec{k}\cdot\vec{r})}$$

$$\begin{cases} \vec{k_i} = |k_i|\vec{e_{ki}} = k_{ix}\vec{e_x} + k_{iy}\vec{e_y} + k_{iz}\vec{e_z} \\ \vec{k_r} = |k_r|\vec{e_{kr}} = k_{rx}\vec{e_x} + k_{ry}\vec{e_y} + k_{rz}\vec{e_z} \\ \vec{k_t} = |k_t|\vec{e_{kt}} = k_{tx}\vec{e_x} + k_{ty}\vec{e_y} + k_{tz}\vec{e_z} \end{cases}$$

相位匹配条件.

$$\begin{cases} E_{it} = E_{iot}\, e^{j(w_i t - \vec{k_i}\cdot\vec{r})} \\ E_{rt} = E_{rot}\, e^{j(w_r t - \vec{k_r}\cdot\vec{r})} \\ E_{tt} = E_{tot}\, e^{j(w_t t - \vec{k_t}\cdot\vec{r})} \end{cases}$$

当 $z=0$ 时. $E_{it} + E_{rt} = E_{tt}$

代入

$$\underline{w_i t - \vec{k_i}\cdot\vec{r} = w_r t - \vec{k_r}\cdot\vec{r} = w_t t - \vec{k_t}\cdot\vec{r}}$$

$$\Rightarrow \begin{cases} \boxed{w_i = w_r = w_t} \\ \vec{k_i}\cdot\vec{r} = \vec{k_r}\cdot\vec{r} = \vec{k_t}\cdot\vec{r} \end{cases}$$

$$|k_i| = w_i\sqrt{\mu_1\varepsilon_1}$$

$$\vec{r} = x\vec{e_x} + y\vec{e_y}$$

$$|k_r| = w_r\sqrt{\mu_1\varepsilon_1} = |k_i|$$

$$k_{ix}x + k_{iy}y = k_{rx}x + k_{ry}y = k_{tx}x + k_{ty}y$$

$$k_{iz} = k_{rz}$$

$$\boxed{\begin{cases} k_{ix} = k_{rx} = k_{tx} \\ k_{iy} = k_{ry} = k_{ty} \end{cases}}$$

反射定律:

$$\frac{\sin\theta_r}{\sin\theta_i} = \frac{\frac{k_{rx}}{|k_r|}}{\frac{k_{ix}}{|k_i|}} = 1 \Rightarrow \theta_r = \theta_i$$

$$n = \sqrt{\mu\varepsilon}$$

折射定律:

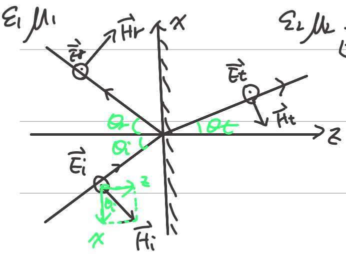
$$\boxed{\frac{\sin\theta_t}{\sin\theta_i} = \frac{\frac{k_{tx}}{|k_t|}}{\frac{k_{ix}}{|k_i|}} = \frac{|k_i|}{|k_t|} = \frac{w_i\sqrt{\mu_1\varepsilon_1}}{w_t\sqrt{\mu_2\varepsilon_2}} = \frac{n_1}{n_2}}$$

反射系数,透射系数

$$\vec{E} = \vec{E}_\perp + \vec{E}_\parallel$$

垂直极化波:

两边都是理想介质



边界条件: $E_{i0} + E_{r0} = E_{t0}$

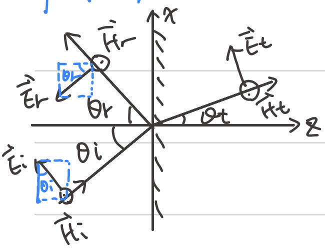
$$-H_{i0}\cos\theta_i + H_{r0}\cos\theta_r = -H_{t0}\cos\theta_t$$

即: $-\dfrac{E_{i0}}{\eta_1}\cos\theta_i + \dfrac{E_{r0}}{\eta_1}\cos\theta_r = -\dfrac{E_{t0}}{\eta_2}\cos\theta_t$

$$\theta_i = \theta_r$$

$$\begin{cases} \Gamma_\perp = \dfrac{E_{r0}}{E_{i0}} = \dfrac{\eta_2\cos\theta_r - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \\ T_\perp = \dfrac{E_{t0}}{E_{i0}} = \dfrac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \end{cases}$$

平行极化波.



边界条件: $\begin{cases} E_{i0}\cos\theta_i - E_{r0}\cos\theta_r = E_{t0}\cos\theta_t \\ H_{i0} + H_{r0} = H_{t0} \end{cases}$

即, $\dfrac{E_{i0}}{\eta_1} + \dfrac{E_{r0}}{\eta_1} = \dfrac{E_{t0}}{\eta_2}$

$$\theta_i = \theta_r$$

$$\Rightarrow \Gamma_\parallel = \frac{E_{r0}}{E_{i0}} = \frac{\eta_1\cos\theta_i - \eta_2\cos\theta_t}{\eta_1\cos\theta_i + \eta_2\cos\theta_t}.$$

$$T_\parallel = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2\cos\theta_i}{\eta_1\cos\theta_i + \eta_2\cos\theta_t}$$