chapter 1: introduction

General model

Analog model

digital model

Advantages of digital communication:

1) High anti-interference ability and a decision-making receiver at the receiver. 2) Error correcting techniques, such as error-correcting coding, can be used. 3) The digital encryption can be used. Therefore, the security of the system is greatly improved. 4) Digital communication equipment: Design and manufacture are easier. Performance index: Efficiency and Reliability.

For analog: effective transmission bandwidth (lower is better), SNR (higher is better).

For digital: Transmission rate / Band utilization $\eta = \frac{R_B}{B}$, symbol/bit error rate. R_B: symbol rate, unit: Baud, $R_B = \frac{1}{T_s}$.

R_b: information rate, unit: bps, $R_b = R_B \times H$. P_e (symbol error rate) $= \frac{\text{received symbols in error}}{\text{total symbols}}$.

P_b (bit error rate) $= \frac{\text{received bits in error}}{\text{total bits}}$.

Binary: $P_b = P_e$. M-ary: $P_b < P_e$.

chapter 2: random process

1) mean

$$E\{x(t)\} = \int_{-\infty}^{+\infty} x f_x(x,t)\, dx = a(t)$$

2) variance

$$D\{x(t)\} = E\big[(x(t) - a(t))^2\big]$$

$$D\{x(t)\} = E[x^2(t)] - a^2(t)$$

3) Auto correlation function

$$R(t_1,t_2) = E\{x(t_1)x(t_2)\}$$

$$R(t_1,t_2) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x_1 x_2 f_2(x_1,x_2,t_1,t_2)\, dx_1 dx_2$$

$$\tau = t_2 - t_1, \qquad R(t_1,t_2) = R(t_1, t_1+\tau)$$

4) Auto covariance function

$$B(t_1,t_2) = E[(x(t_1) - a(t_1))(x(t_2) - a(t_2))]$$

$$B(t_1,t_2) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (x_1 - a(t_1))(x_2 - a(t_2)) f_2\, dx_1 dx_2$$

$$B(t_1,t_2) = R(t_1,t_2) - a(t_1)a(t_2)$$

5) cross correlation function

$$R_{xy}(t_1,t_2) = E\{x(t_1)y(t_2)\}$$

6) cross covariance function

$$B_{xy}(t_1,t_2) = E[(x(t_1) - a_x(t_1))(y(t_2) - a_y(t_2))]$$

stationary random process (wide-sense-stationary, WSS).

properties: 1) mean is a constant; 2) auto correlation function depends only on the time interval τ.

ergodicity proof: 1) judge whether stationary (using statistical average); 2) compute time average; 3) compare them.

The auto correlation function of WSS.

(1) $R(0) = E[\xi^2(t)] = S$ — average power

(2) $R(\infty) = E^2[\xi(t)] = a^2$ — DC power

(3) $R(0) - R(\infty) = \sigma^2$ — AC power (variance)

(4) $R(\tau) = R(-\tau)$ — even function

(5) $|R(\tau)| \le R(0)$ — upper bound

Power spectral density (PSD): $P(f) = \lim_{T\to\infty} \frac{1}{T} |S_T(f)|^2$

Average power:

$$P = \int_{-\infty}^{\infty} P(f)\, df = \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} |S_T(f)|^2\, df$$

Autocorrelation and PSD:Fourier transform relationship:

$$P(f) = \int_{-\infty}^{\infty} R(\tau)\, e^{-j2\pi f\tau}\, d\tau, \qquad R(\tau) = \int_{-\infty}^{\infty} P(f)\, e^{j2\pi f\tau}\, df$$

Stationary random process through LTI system:

1) Mean (constant): if input mean is a, $E\{y(t)\} = a\, H(0)$.

2) ACF: $R_y(t_1, t_1+\tau) = R_y(\tau)$.

3) PSD: $P_y(f) = |H(f)|^2 P_x(f), \quad P_y(f) \longleftrightarrow R_y(\tau)$

Narrowband random process

Representation 1: $x(t) = a_\xi(t)\cos[\omega_c t + \varphi_\xi(t)], \quad a_\xi(t) \ge 0$

$a_\xi(t)$: random envelope, $\varphi_\xi(t)$: random phase. Representation 2: $x(t) = \xi_c(t)\cos\omega_c t - \xi_s(t)\sin\omega_c t$

$\xi_c(t)$: in-phase component, $\xi_s(t)$: quadrature component.

Relationship: $a_\xi(t) = \sqrt{\xi_c^2(t) + \xi_s^2(t)}$,

$\varphi_s(t) = \arctan\big(\xi_s(t)/\xi_c(t)\big)$. Properties for stationary Gaussian narrowband process

Assume $E\{\xi(t)\} = 0$, $D\{\xi(t)\} = \sigma^2$. Then

$E\{\xi_c(t)\} = E\{\xi_s(t)\} = 0, \quad D\{\xi_c(t)\} = D\{\xi_s(t)\} = \sigma^2$,

$\xi_c(t)$ and $\xi_s(t)$ are uncorrelated. $R_{cs}(0) = 0$ Envelope and phase statistics Joint pdf of in-phase and quadrature:

$f(a_\xi, \varphi_\xi) = \frac{a_\xi}{2\pi\sigma_\xi^2}\exp\left[-\frac{a_\xi^2}{2\sigma_\xi^2}\right]$.

Envelope(Rayleigh): $f(a_\xi) = \frac{a}{\sigma_s^2}\exp\left(-\frac{a^2}{2\sigma_s^2}\right), \quad a \ge 0.$

Phase(uniform): $f_(\varphi_\xi) = \frac{1}{2\pi}, \quad 0 < \varphi \le 2\pi.$

sinusoid is added to the narrowband Gaussian noise, the envelope $Z(t)$ is Rician distributed.

$r(t) = A\cos(\omega_c t + \theta) + n(t) = z_c(t)\cos\omega_c t + z_s(t)\sin\omega_c t = z(t)\cos[\omega_c t + \varphi(t)]$

Gaussian white noise

Power spectral density (PSD):

$$P_n(f) = \frac{n_0}{2}, \qquad -\infty < f < +\infty$$

Autocorrelation function (ACF): $R_n(\tau) = \frac{n_0}{2}\,\delta(\tau)$

Low-pass white noise

PSD:

$$P_n(f) = \begin{cases} \frac{n_0}{2}, & |f| \le f_H, \\ 0, & \text{otherwise}, \end{cases}$$

$$R_n(\tau) = n_0 f_H\, \frac{\sin(2\pi f_H \tau)}{2\pi f_H \tau}$$

Band-pass white noise

PSD:

$$P_n(f) = \begin{cases} \frac{n_0}{2}, & f_c - \frac{B}{2} \le |f| \le f_c + \frac{B}{2}, \\ 0, & \text{otherwise}, \end{cases}$$

ACF:

$$R_n(\tau) = n_0 B\, \frac{\sin(\pi B\tau)}{\pi B\tau}\cos(2\pi f_c \tau)$$

Noise power in the band: $N = n_0 B$ Channel capacity: maximum average information rate for error–free transmission.

1)Nyquist theorem

$$C = 2B\log_2 M \quad \text{(bits/s)}.$$

2) Shannon capacity (AWGN channel, error–free)

$$C = B\log_2\left(1 + \frac{S}{N}\right) \quad \text{(bits/s)},$$

N_0 the one–sided noise power spectral density. $S \to \infty$, $N_0 \to 0 \;\Rightarrow\; C \to \infty; B \to \infty \;\Rightarrow\; C \to 1.44\,\frac{S}{N_0}$.

Modulation theorem: $f(t)\cos\omega_c t \iff \frac{1}{2}\big[F(\omega - \omega_c) + F(\omega + \omega_c)\big]$.

AM: Time domain: $s_{AM}(t) = [A_0 + m(t)]\cos\omega_c t$

standard form: $s_{AM}(t) = A_c[1 + k_a m(t)]\cos\omega_c t$

k_a: amplitude sensitivity

Frequency domain: $S_{AM}(\omega) = \pi A_0[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)] + \frac{1}{2}\left[M(\omega-\omega_c)+M(\omega+\omega_c)\right]$.
Envelope–detection condition: $|m(t)|_{\max} \le A_0$.
bandwidth: $B_{AM} = 2f_m$

Modulation index

Define $M = |m(t)|_{\max}/A_0$.
For single–tone modulation $m(t) = A_m\cos\omega_m t$ we have $\overline{m^2(t)} = A_m^2/2$ and $\eta_{AM} = M^2/(2+M^2)$, which is maximized when M is near 1.

DSB–SC

Time domain: $s_{DSB}(t) = m(t)\cos\omega_c t$.
Spectrum: $S_{DSB}(\omega) = \frac{1}{2}\left[M(\omega-\omega_c)+M(\omega+\omega_c)\right]$.
Bandwidth: $B_{DSB} = 2f_m$.
The power efficiency of DSB is 100%.

SSB

Concept: start from DSB and suppress one sideband.
Filtering method: pass $s_{DSB}(t)$ through a band–pass filter $H_{SSB}(\omega)$ that keeps only USB or LSB.
Ideal USB filter: $H_{USB}(\omega) = 1$ for $|\omega| > \omega_c$ and 0 for $|\omega| \le \omega_c$ (around the lower sideband); ideal LSB filter is complementary.
Phase–shifting (phasing) method for a general message $m(t)$ uses its Hilbert transform $\hat{m}(t)$: $s_{USB}(t) = \frac{1}{2}\left[m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t\right]$ and $s_{LSB}(t) = \frac{1}{2}\left[m(t)\cos\omega_c t + \hat{m}(t)\sin\omega_c t\right]$.
The Hilbert–transform filter has frequency response $H_H(\omega) = -j\,\mathrm{sgn}(\omega)$.
SSB bandwidth: $B_{SSB} = f_m$ (half that of DSB).

VSB (vestigial–sideband) modulation

Start from SSB and keep a vestige of the other sideband: bandwidth satisfies $f_m < B_{VSB} < 2f_m$.
Typical application: video transmission (TV).
If $S_{VSB}(\omega) = S_{DSB}(\omega)H(\omega)$, then after synchronous demodulation the baseband response is proportional to $H(\omega+\omega_c)+H(\omega-\omega_c)$; for distortionless detection this sum should be approximately constant for $|\omega| \le \omega_m$.

AM envelope detection: model

Signal at BPF output: $s_m(t) = [A_0+m(t)]\cos\omega_c t$, average power $S_i = \overline{s_m^2(t)} = \frac{1}{2}A_0^2 + \frac{1}{2}P_m$.
Noise at BPF output: $n_i(t) = n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t$, power $N_i = n_0 B$.
Input to the envelope detector: $x(t) = s_m(t)+n_i(t) = [A_0+m(t)+n_c(t)]\cos\omega_c t - n_s(t)\sin\omega_c t = E(t)\cos[\omega_c t+\varphi(t)]$, where $E(t) = \sqrt{[A_0+m(t)+n_c(t)]^2+n_s^2(t)}$ and $\varphi(t) = \arctan\big(n_s(t)/[A_0+m(t)+n_c(t)]\big)$.

Large SNR case ($[A_0+m(t)] \gg \sqrt{n_c^2(t)+n_s^2(t)}$): $E(t) \approx A_0+m(t)+n_c(t)$, so the output signal power is $S_o = \overline{m^2(t)} = P_m$, the output noise power is $N_o = n_0 B$, and the AM modulation gain is $G_{AM} = (S_o/N_o)/(S_i/N_i) = 2P_m/(A_0^2+P_m) \le 1$ (since $|m(t)|_{\max} \le A_0$; for $A_0^2 \approx 2P_m$ we get $G_{AM} \approx 2/3$).

Small SNR case (envelope detector)

Condition: $[A_0+m(t)] \ll \sqrt{n_s^2(t)+n_c^2(t)}$.
Let $R(t) = \sqrt{n_c^2(t)+n_s^2(t)}$ and $\cos\theta(t) = \dfrac{n_c(t)}{R(t)}$.
Then the envelope is $E(t) = R(t) + [A_0+m(t)]\cos\theta(t)$.
The relation between output SNR and input SNR exhibits a threshold effect: Below a certain carrier–to–noise ratio (threshold), the SNR of a nonlinear envelope detector deteriorates rapidly; A coherent detector has no such threshold.

Coherent detection in DSB $s_{DSB}(t) = m(t)\cos(\omega_c t)$

Modulation gain in DSB: Input noise: $N_i = n_0 B$, Input signal: $S_i = \overline{s_m^2(t)} = \frac{1}{2}P_m$; Output noise: $N_o = \frac{1}{4}Bn_0$, Output signal: $S_o = \overline{m_o^2(t)} = \frac{1}{4}P_m$; Bandwidth: $B = 2f_m$;Gain:
$G_{DSB} = \frac{(\mathrm{SNR})_o}{(\mathrm{SNR})_i} = 2$.

Angle modulation (PM / FM)

General form: $s(t) = A\cos[\omega_c t+\varphi(t)]$,
PM: $\varphi(t) = k_p m(t)$, where k_p is phase sensitivity (rad/V).
FM: $\dfrac{d\varphi(t)}{dt} = k_f m(t)$, where k_f is frequency sensitivity (rad/s/V).
PM signal: $s_{PM}(t) = A\cos[\omega_c t + k_p m(t)]$.
FM signal: $s_{FM}(t) = A\cos\left[\omega_c t + k_f\int_0^t m(\tau)d\tau\right]$.
The instantaneous frequency is $f(t) = f_c + \Delta f\cos(2\pi\times f_m t)$, the angular frequency is $\omega(t) = 2\pi f(t)$, the total phase is $\theta(t) = \int_{-\infty}^t \omega(\tau)d\tau$, and the FM signal expression is $S_{\mathrm{FM}}(t) = A\cos\theta(t)$.

If only $s(t)$ is known and $m(t)$ is unknown, one cannot distinguish PM from FM.

Single–tone modulation $m(t) = A_m\cos\omega_m t$

PM: $\varphi(t) = k_p A_m\cos\omega_m t$, modulation index $m_p = k_p A_m$, so $s_{PM}(t) = A\cos[\omega_c t + m_p\cos\omega_m t]$.
FM: $d\varphi(t)/dt = k_f A_m\cos\omega_m t$ gives $\varphi(t) = \dfrac{k_f A_m}{\omega_m}\sin\omega_m t = m_f\sin\omega_m t$, with $m_f = \dfrac{k_f A_m}{\omega_m} = \dfrac{\Delta f}{f_m}$ (FM index, Δf peak deviation), so $s_{FM}(t) = A\cos[\omega_c t + m_f\sin\omega_m t]$.

Narrowband FM (NBFM)

FM signal: $s_{FM}(t) = A\cos[\omega_c t + k_f\int_0^t m(\tau)d\tau]$.
If $k_f\int_0^t |m(\tau)|d\tau \ll 1$ (small phase deviation), then $s_{NBFM}(t) \approx A\cos\omega_c t - Ak_f\int_0^t m(\tau)d\tau\,\sin\omega_c t$.
$S_{NBFM}(\omega) \approx \pi A[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)] + \dfrac{Ak_f}{2}[\dfrac{M(\omega+\omega_c)}{\omega+\omega_c} - \dfrac{M(\omega-\omega_c)}{\omega-\omega_c}]$.

Wideband FM (WBFM), single–tone

With $m(t) = A_m\cos\omega_m t$, $s_{FM}(t) = A\cos[\omega_c t + m_f\sin\omega_m t]$ can be expanded as $s_{FM}(t) = A\sum_{n=-\infty}^{\infty} J_n(m_f)\cos[(\omega_c+n\omega_m)t]$, so the spectrum has a carrier plus infinite sidebands at $\omega_c \pm n\omega_m$.
Carson rule for approximate FM bandwidth: $B_{FM} = 2(m_f+1)f_m = 2(\Delta f + f_m)$. Total FM power: $P_{FM} = \overline{s_{FM}^2(t)} = \dfrac{A^2}{2}$

FM demodulation (slope / discriminator)

Differentiator–type discriminator: input $s_{FM}(t) = A\cos[\omega_c t + k_f\int_0^t m(\tau)d\tau]$, output after differentiation: $s_d(t) = \dfrac{d}{dt}s_{FM}(t) = -A[\omega_c + k_f m(t)]\sin[\omega_c t + k_f\int_0^t m(\tau)d\tau]$.
After a band–pass filter and envelope detection, amplitude is $\propto \omega_c + k_f m(t)$; subtracting the DC term and low–pass filtering yields $m_o(t) \approx k_d k_f m(t)$, where k_d is the discriminator gain.

FM demodulators can be noncoherent (discriminator, for NBFM and WBFM) or coherent (for NBFM).

Anti–noise performance of FM

Model: $s_{FM}(t)+n(t) \to$ BPF $\to$ limiter $\to$ discriminator $\to$ LPF.
High SNR: FM SNR gain $G_{FM} = 3m_f^2(m_f+1)$ with $B_{FM} = 2(m_f+1)f_m \approx 2(\Delta f + f_m)$, so wideband FM (large m_f) has strong anti–noise capability at the expense of bandwidth.
Low SNR: a threshold effect

FM is also widely used in frequency–division multiplexing and frequency shifting.