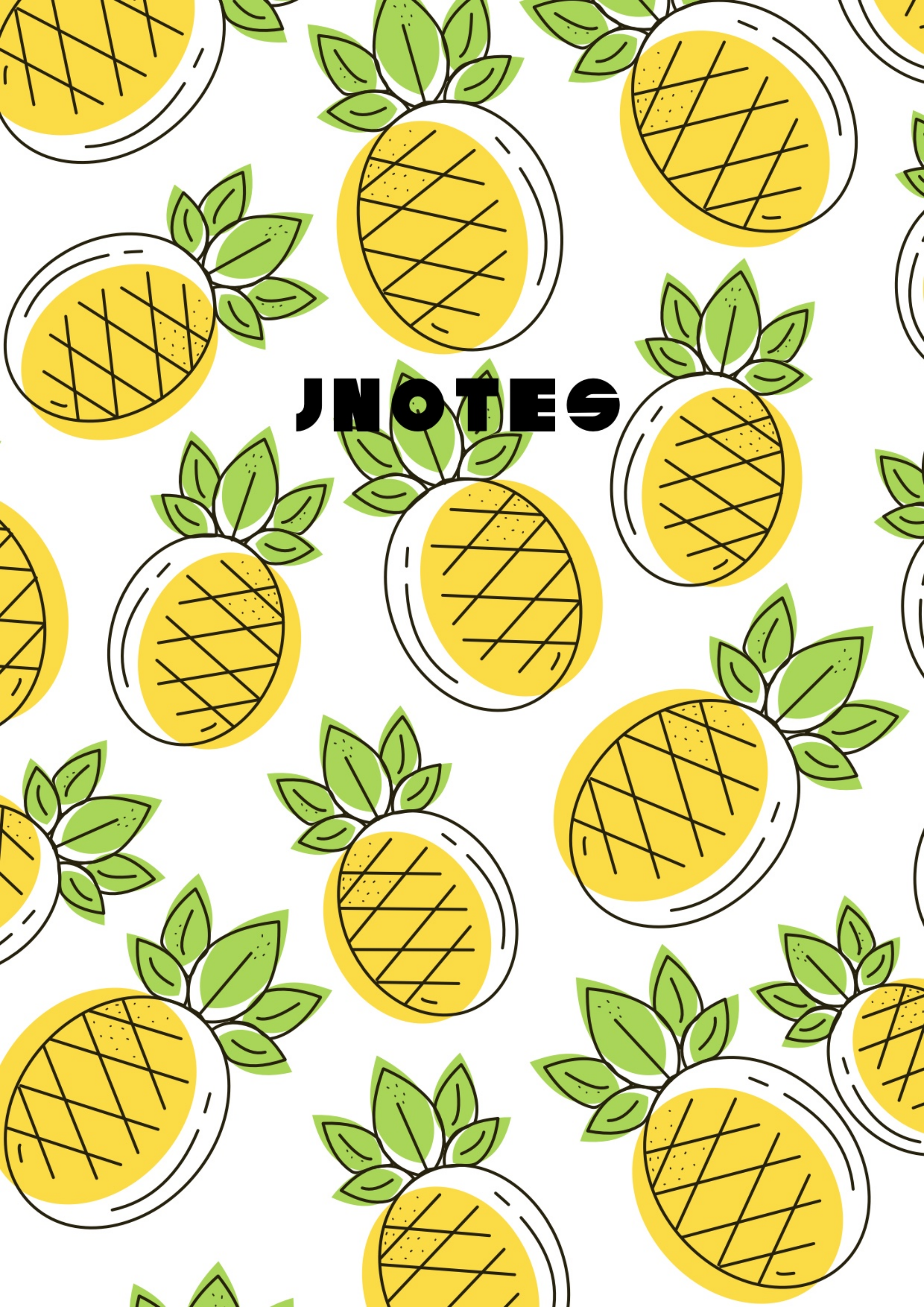

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信息 Information : Effective content of Message

消息 Message: the carrier of information (载体) 语音.文字.图像

信号 signal : the carrier of Message 电信号/光信号

通信 communication. Transmit information (信息) in message (消息) by electrical signal (电信号)

Analog ~ 信号参量取值连续 continuous.

digital ~ 离散 discrete

General Model:

Source of information → Transmitter → channel. → Receiver → User of Information

↑ noise.

Analog:

模拟信源 (模拟消息) →(基带信号) 调制器 (Modulator) →(带通信号) 信道 (↑ 噪声) →(带通信号) 解调器 (Demodulator) →(基带信号) 信宿 (模拟消息)

模拟信号 <=> 原始电信号 (基带信号) by 信源/信宿

基带信号 <=> 带通信号 (已调信号) by Modulator/Demodulator

Digital:

信源 → Transmitter [信源编码 → 加密 → 信道编码 → 数字调制] → 信道 (↑ 噪声源) → Receiver [数字解调 → 信道译码 → 解密 → 信源译码] → 信宿

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编码.译码.调制.解调.同步.

Advantage. ①抗干扰能力强 ②传输差错可控 ③数字加密.安全性高 ④设备易于集成

通信系统分类
- 模式:
  - 传输方向:单工、半双工、全双工
  - 码元排列:串行、并行
- 调制方式:
  - CW (sine)
    - 数字. ASK. FSK. PSK → 非线性
    - 模拟. 线性调制. 角调(PM. FM)
      - ↳ AM. SSB. DSB. VSB
  - PW (rect)
    - 数字:PCM
    - 模拟. PAM. PDM PPM
- 复用
  - FDM 频分
  - TDM 时分
  - CDM 码分

频段:长波.短波.微波.激光

信息:不确定性描述 $I = \log_a \frac{1}{p(x)} = -\log_a p(x)$

$a=2$ 比特 b

$a=e$ 奈特

$a=10$ 哈特莱

M进制等概:$I = \log_2 M$

信息熵 $H(x)$ (平均信息量) $H = \sum_{i=1}^{M} p(x_i)\log_2 \frac{1}{p(x_i)}$

等概时信息熵最大 $\max H = \log_2 M$

performance index
- 有效性 (Efficiency) speed
- 可靠性 (Reliability) quality

analog:
- bandwidth (有效传输带宽)
- SNR

频带利用率.

$R_B = \frac{1}{T_s}$

digital:
- Transmission rate / Band utilization $\eta = \frac{R_B}{B}$
- symbol / Bit error rate
  误码率 / 误信率.

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R_B: symbol rate (传码率 / 波特率) Unit: Baud.

R_b: Information rate (传信率) Unit: bit/s. b/s. bps

$R_b = R_B \cdot H \xrightarrow{\text{等概}} R_b = R_B \cdot \log_2 M$.

P_e: 误码率 symbol error rate $\frac{\text{received symbols in error}}{\text{total symbols}}$

P_b: 误信率 Bit error rate $\frac{\text{received bits in error}}{\text{total bits}}$

二进制: $P_b = P_e$ M-ary: $P_b < P_e$

Random Process.

$\xi(t) = \{\xi_1(t)\ \xi_2(t) \cdots \xi_n(t)\}$

Definition: 样本函数集合 / 不同时间随机变量的集合.

$\xi(t) = \{\xi(t_k)\}$.

$\xi(t_k) = \{\xi_i(t_k),\ i = 1, 2, 3 \cdots n\}$.

描述方法:
- 分布函数 $F(x)$
- pdf $f(x)$
- 数字特征

$E(X) = \int_{-\infty}^{+\infty} x f(x) dx \quad E[g(x)] = \int_{-\infty}^{+\infty} g(x) f(x) dx$

一维 $F_1(x_1, t_1) = P[\xi(t_1) \le x_1]$

pdf: $\frac{\partial F_1(x_1, t_1)}{\partial x_1} = f_1(x_1, t_1)$

二维: $F_2(x_1, x_2; t_1, t_2) = P\{\xi(t_1) \le x_1, \xi(t_2) \le x_2\}$

$\frac{\partial^2 F_2(x_1, x_2; t_1, t_2)}{\partial x_1 \partial x_2} = f_2(x_1, x_2; t_1, t_2)$

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1) Mean (均值) — 摆动中心

$$E[\xi(t)] = \int_{-\infty}^{+\infty} x f_1(x,t)\,dx = a(t)$$

2) Variance (方差) — 偏离程度:

$$D[\xi(t)] = E\{[\xi(t) - a(t)]^2\}$$

$$D[\xi(t)] = E[\xi^2(t)] - a^2(t)$$

平方的均值 — 均值的平方

3) Autocorrelation Function (自相关函数) — 同一过程的关联程度.

$$R(t_1,t_2) = E[\xi(t_1)\xi(t_2)]$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x_1 x_2 f_2(x_1,x_2;t_1,t_2)\,dx_1 dx_2$$

令 $\tau = t_2 - t_1$, 则有 $R(t_1,t_2) = R(t_1, t_1+\tau)$

4) Autocovariance Function (自协方差函数)

$$B(t_1,t_2) = E\{[\xi(t_1) - a(t_1)][\xi(t_2) - a(t_2)]\}$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} [x_1 - a(t_1)][x_2 - a(t_2)] f_2(x_1,x_2;t_1,t_2)\,dx_1 dx_2$$

$$B(t_1,t_2) = R(t_1,t_2) - a(t_1)a(t_2)$$

5) Cross-Correlation Function (互相关函数) — 两个过程的关联程度.

$$R_{\xi\eta}(t_1,t_2) = E[\xi(t_1)\eta(t_2)]$$

6) Cross-covariance Function (互协方差函数)

$$B_{\xi\eta}(t_1,t_2) = E\{[\xi(t_1) - a_\xi(t_1)][\eta(t_2) - a_\eta(t_2)]\}$$

Stationary Random process

1. 严平稳 (~ in the strict sense)

有限维分布与时间起点无关

property: ① 均值为常数 ② 自相关函数只与时间间隔τ有关.

2. 宽平稳 (wide-sense stationary)

satisfies: ① Mean = Constant

② Autocorrelation function is only related to time interval τ.

各态历经性 (Ergodicity):

其数字特征可由随机过程的任一实现的时间平均值来替代

ergodic $\underset{\times}{\overset{\checkmark}{\rightleftarrows}}$ stationary

to prove Ergodicity: ① judge whether stationary → 求统计平均.

② 求时间平均

③ 比较是否相等

$$\overline{a} = \overline{X(t)} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,dt$$

用时间平均替代统计平均

$$\overline{R(\tau)} = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)x(t+\tau)\,dt$$

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The auto-correlation function of WSS

$R(\tau)=E[\xi(t)\xi(t+\tau)]$

(1) $R(0)=E[\xi^2(t)]=S$ — 平均功率.

$\xi(t)$ 与 $\xi(t+\infty)$ 是独立的

(2) $R(\infty)=E^2[\xi(t)]=a^2$ —直流功率

$E[\xi(t)]\cdot E[\xi(t+\infty)]=E^2[\xi]=a^2$

(3) $R(0)-R(\infty)=S-a^2$ — 交流功率（方差）

(4) $R(\tau)=R(-\tau)$ 偶函数

(5) $|R(\tau)|<R(0)$ 上界

PSD 功率谱密度.

能量信号

能量谱密度 J/Hz

$s(t)\Leftrightarrow S(f)$ Parseval Th. $E=\int_{-\infty}^{+\infty}s^2(t)dt=\int_{-\infty}^{+\infty}\boxed{|S(f)|^2}df$

功率信号. $S_T(t)=\begin{cases}s(t) & |t|\le\frac{T}{2}\\ 0 & \text{otherwise}\end{cases}$ 能量. $\int_{-T/2}^{T/2}S_T^2(t)dt=\int_{-\infty}^{+\infty}|S_T(f)|^2df$

$P(f)=\lim\limits_{T\to\infty}\frac{1}{T}|S_T(f)|^2$ $P=\int_{-\infty}^{+\infty}P(f)df=\lim\limits_{T\to\infty}\frac{1}{T}\int_{-\infty}^{+\infty}|S_T(f)|^2df$ Power

PSD

$P(f)=\int_{-\infty}^{+\infty}R(\tau)e^{-j2\pi f\tau}d\tau$ $R(\tau)=\int_{-\infty}^{+\infty}P(f)e^{j2\pi f\tau}df$

维纳-辛钦定理: $P(f)$ 与 $R(\tau)$ 是一对傅里叶变换

PSD Auto-correlation function

当 $\tau=0$ 时. $R(0)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}P_\xi(w)dw=\int_{-\infty}^{+\infty}P_\xi(f)df$

平均功率 功率谱密度积分

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if the WSS is ergodic:

PSD properties: ① $P_\xi(\omega) \geq 0$ ② $P_\xi(-\omega) = P_\xi(\omega)$.

Gaussian process

Definition: 所有有限维分布都是正态分布.

properties:

1) WSS + Gaussian $\longrightarrow$ strict-sense stationary

2) 不相关即统计独立

① independent: $f(x_i, x_j) = f(x_i)\cdot f(x_j)$

② Uncorrelated: $C_x(t_i, t_k) = 0$　　协方差函数为0 ⟺ 不相关

$C_x(t_i, t_k) = E\{[X(t_i) - \mu_X(t_i)][X(t_k) - \mu_X(t_k)]\}$, $i \neq k$

Uncorrelated + Gaussian $\longrightarrow$ independent

independent $\longrightarrow$ uncorrelated

时间k上的高斯随机变量的一维概率密度函数

$$f_n(x_1, x_2, \dots, x_n; t_1, t_2 \dots t_n) = \prod_{k=1}^{n} \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left[-\frac{(x_k - a_k)^2}{2\sigma_k^2}\right]$$

3) 线性变换后仍然为 Gaussian process.

$$\text{G.P.} \longrightarrow \boxed{\text{Linear System}} \longrightarrow \text{G.P.}$$

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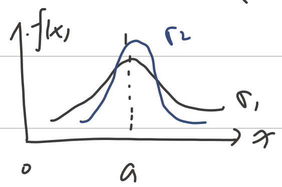
高斯随机变量:

PDF: $f(x)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-a)^2}{2\sigma^2}\right)$ denoted as $N(a,\sigma^2)$



distribution function.

$$F(x)=P(\xi\le x)=\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{(x-a)^2}{2\sigma^2}\right]$$

误差函数: $\mathrm{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$ $\quad\mathrm{erf}(0)=0$, $\mathrm{erf}(\infty)=1$

补 ~ : $\mathrm{erfc}(x)=\frac{2}{\sqrt{\pi}}\int_x^{\infty}e^{-t^2}dt$ $\quad\mathrm{erfc}(0)=1$, $\mathrm{erfc}(\infty)=0$

$\mathrm{erfc}(x)=1-\mathrm{erf}(x)$

$$F(x)=\begin{cases}\frac{1}{2}+\frac{1}{2}\mathrm{erf}\left(\frac{x-a}{\sqrt{2}\sigma}\right), & x\ge a\\ 1-\frac{1}{2}\mathrm{erfc}\left(\frac{x-a}{\sqrt{2}\sigma}\right) & x<a\end{cases}$$

Stationary Random process through LTI system.

1) Mean. Constant $E[\xi_o(t)]=a\cdot H(0)$

2) ACF. $R_o(t_1,t_1+\tau)=R_o(\tau)$

3) PSD: $P_o(f)=|H(f)|^2\cdot P_i(f)$

$P_o(f)\longleftrightarrow R_o(\tau)$ Fourier Transform input: stationary / Gaussian output: stationary / Gaussian.

Narrowband random process:

Conditions: $\Delta f \ll f_c$ 且 f_c 远离零频率. $\begin{cases} \Delta f \ll f_c \\ f_c \gg 0 \end{cases}$

Expressions:

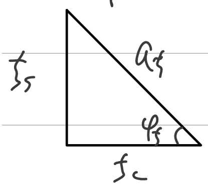
1) $\xi(t) = a_\xi(t) \cos[w_c t + \varphi_\xi(t)]$. $a(\xi) \geq 0$ 包络/相位

Random envelop Random phase

2) $\xi(t) = \xi_c(t) \cos w_c t - \xi_s(t) \sin w_c t$ 同相/正交.

In-phase component Quadrature component

Relationship:



property: Assume $E[\xi(t)] = 0$ $D[\xi(t)] = \sigma^2$

1) 同相-正交 $\xi(t) = \xi_c(t) \cos w_c t - \xi_s(t) \sin w_c t$

$$E[\xi(t)] = E[\xi_c(t)] \cos w_c t - E[\xi_s(t)] \sin w_c t$$

$\because E[\xi(t)] = 0$ $\therefore E[\xi_c(t)] = E[\xi_s(t)] = 0$

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ACF: $R_\xi(t,t+\tau)=E[\xi(t)\xi(t+\tau)]$ 代入

$=R_c(t,t+\tau)\overset{1}{\cancel{\cos\omega_c t}}\cos\omega_c(t+\tau)-R_{cs}(t,t+\tau)\overset{1}{\cancel{\cos\omega_c t}}\sin\omega_c(t+\tau)$

$-R_{sc}(t,t+\tau)\overset{=0}{\underline{\sin\omega_c t}}\cos\omega_c(t+\tau)+R_s(t,t+\tau)\overset{=0}{\underline{\sin\omega_c t}}\sin\omega_c(t+\tau)$

又∵ $\xi(t)$ is stationary　令 $t=0$

∴ $R_\xi(\tau)=R_c(t,t+\tau)\cos\omega_c\tau-R_{cs}(t,t+\tau)\sin\omega_c(t+\tau)$

$=\underline{R_c(\tau)}\cos\omega_c\tau-\underline{R_{cs}(\tau)}\sin\omega_c\tau$

令 $t=\pi/2\omega_c$

$R_\xi(\tau)=\underline{R_s(\tau)}\cos\omega_c\tau+\underline{R_{sc}(\tau)}\sin\omega_c\tau$

$\Rightarrow R_c(\tau)=R_s(\tau)$

$R_{cs}(\tau)=-R_{sc}(\tau)$

∴ $\xi(t)$ is stationary $\longrightarrow$ $\xi_s(t)$ and $\xi_c(t)$ is stationary

又: $R_{cs}(\tau)=R_{sc}(-\tau)$　∴ $R_{sc}(\tau)=-R_{sc}(-\tau)$　奇函数 $\begin{cases}R_{sc}(0)=0\\R_{cs}(0)=0\end{cases}$

$R(0)=E[\xi^2(t)]$　令 $\tau=0$ 代入

$R_\xi(0)=R_c(0)=R_s(0)$　∴ $\sigma_\xi^2=\sigma_c^2=\sigma_s^2$

∴ $\xi(t)$, $\xi_c(t)$, $\xi_s(t)$ 有相同的平均功率与方差

Conclusion: $\xi(t)=\xi_c(t)\cos\omega_c t-\xi_s(t)\sin\omega_c t$　(Stationary & Gaussian.

$E[\xi(t)]=0$, $D[\xi(t)]=\sigma^2$ $\Rightarrow$ $\xi_c(t)$与$\xi_s(t)$　S & G

均值为0. 方差相等: $\sigma_\xi^2=\sigma_c^2=\sigma_s^2$　　$R_{cs}(0)=0$ 不相关 → 统计独立

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Joint PDF. $f(\xi_c, \xi_s) = f(\xi_c)\cdot f(\xi_s) = \frac{1}{2\pi\sigma_\xi^2}\exp[-\frac{\xi_c^2+\xi_s^2}{2\sigma_\xi^2}]$

$$f(a_\xi, \varphi_\xi) = f(\xi_c, \xi_s)\left|\frac{\partial(\xi_c, \xi_s)}{\partial(a_\xi, \varphi_\xi)}\right| = a_\xi f(\xi_c, \xi_s)$$

$\Rightarrow f(a_\xi, \varphi_\xi) = \frac{a_\xi}{2\pi\sigma_\xi^2}\exp[-\frac{a_\xi^2}{2\sigma_\xi^2}]$ ～ Rayleigh 分布

包络 envelop. $f(a_\xi) = \int_{-\infty}^{+\infty} f(a_\xi, \varphi_\xi)\,d\varphi_\xi = \frac{a_\xi}{\sigma_\xi^2}\exp[-\frac{a_\xi^2}{2\sigma_\xi^2}]$ $a_\xi \geq 0$

phase. 相位 $f(\varphi_\xi) = \int_{-\infty}^{+\infty} f(a_\xi, \varphi_\xi)\,d a_\xi = \frac{1}{2\pi}$ $0 \leq \varphi_\xi \leq 2\pi$

△ 包络与相位统计独立 ～均匀分布 (uniform)

正弦波加窄带高斯噪声.

$r(t) = A\cos(\omega_c t + \theta) + n(t)$ → $(0, \sigma^2)$

θ: 随机相位 uniformly $(0, 2\pi)$

$= Z_c(t)\cos\omega_c t - Z_s\sin\omega_c t$ $\quad Z_c(t) = A\cos\theta + n_c(t)$

$Z_s(t) = A\sin\theta + n_s(t)$

$= Z(t)\cos[\omega_c t + \varphi(t)]$

Conclusion: $Z(t)$ 服从莱斯 (Rician) 分布.

$A \to 0$: 退化为 Rayleigh 分布 $\quad A \to \infty$ 退化为 Gaussian 分布

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Gaussian White noise.

$P_n(f) = \frac{n_0}{2} \quad -\infty < f < +\infty$

$\downarrow$ W-H T.h.

$R_n(\tau) = \frac{n_0}{2}\delta(\tau) \Rightarrow$ 当 $\tau \neq 0$ 时 $R(\tau) = 0 \Rightarrow$ 不同时刻的不相关

Low-pass white noise

$P_n(f) = \begin{cases} \frac{n_0}{2} & |f| \le f_H \\ 0 & \text{others} \end{cases} \xLeftrightarrow{\text{W.H.T.h.}} R(\tau) = n_0 f_H \frac{\sin 2\pi f_H \tau}{2\pi f_H \tau}$

Band-pass white noise.

$P_n(f) = \begin{cases} \frac{n_0}{2} & f_c - \frac{B}{2} \le |f| \le f_c + \frac{B}{2} \\ 0 & \text{otherwise.} \end{cases}$ $\rightarrow B \ll f_c$ 窄带高斯白噪声

$\xLeftrightarrow{\text{W.H.T.h.}} R(\tau) = n_0 B \frac{\sin \pi B \tau}{\pi B \tau} \cos 2\pi f_c \tau$

平均功率: $N = n_0 B$ 带宽或称矩形带宽

channel.

- 狭义信道:
  - 有线 → 明线 / 同轴电缆 / 光纤　时不变.
  - 无线 → 时变 / 时不变　$\frac{\text{LOS}}{\text{NLOS}}$ → $h = \frac{D^2}{8r} \approx \frac{D^2}{50}$
- 广义信道: 调制/编码信道

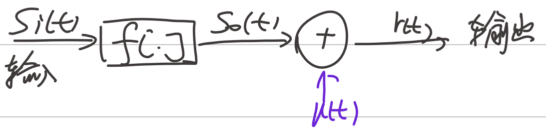
无线信道:

① 电磁波特性: 速度 c. $\lambda = \frac{c}{f}$. 天线尺寸 $h \geq \frac{\lambda}{10}$　Antenna size:

信道的数学模型:

调制信道模型(连续)

叠加噪声线性时变/时不变网络.



$r(t) = f[Si(t)] + n(t) = S_o(t) + n(t)$　{ 恒参信道 / 随参信道

恒参信道:

$S_o(t) = c(t) * Si(t)$

$S_o(w) = C(w)\, Si(w)$ ← 乘性干扰

随参信道:

$f[\cdot] = c(t, \tau)$ ↕ $C(w, \tau)$

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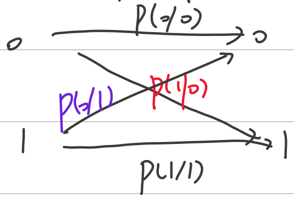
Encoder out ~ Decoder in

编码信道模型（离散）. 范围: 编码器输出 ~ 解码器输入

描述: 转移概率 $p(y|x)$



$P_e = P(0)P(1/0) + P(1)P(0/1)$

$P(0/0) = 1 - P(1/0)$

$P(1/1) = 1 - P(0/1)$

信道特性对传输影响:

① 恒参信道 → { 幅频特性 $|H(\omega)|$. 相频特性 $\varphi(\omega)$.

理想信道: 无失真传输（幅频为常数. 相频是原点直线）

distortion-free transmission.

$H(\omega) = K e^{-j\omega t_d}$

$$\begin{cases} |H(\omega)| = K \\ \varphi(\omega) = \omega t_d \end{cases} \Rightarrow \tau(\omega) = \frac{d\varphi(\omega)}{d\omega} = t_d$$

群时延.

$h(t) = K\delta(t - t_d)$

② 随参信道

特性: 衰减/时延随时间快速变化.

多径效应 (Multipath propagation):

单音信号: $s(t) = A\cos\omega_c t$

Narrow Band (合成窄带信号

$$r(t) = \sum_{i=1}^{n} a_i(t)\cos\omega_c[t - \tau_i(t)]$$

$$= \sum_{i=1}^{n} a_i(t)\cos[\omega_c t + \varphi_i(t)]$$

$\varphi_i(t) = -\omega_c\tau_i(t)$

$$= X(t)\cos\omega_c t - Y(t)\sin\omega_c t$$

$$\begin{cases} X(t) = \sum_{i=1}^{n} a_i(t)\cos\varphi_i \\ Y(t) = \sum_{i=1}^{n} a_i(t)\sin\varphi_i \end{cases}$$

$$= V(t)\cos[\omega_c t + \varphi(t)]$$

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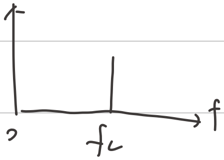
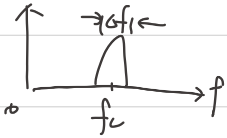
$s(t)=A\cos\omega_0 t$　　$r(t)=V(t)\cos[\omega_c t+\varphi(t)]$





narrowband signal.

两径: 衰落 k　时延 τ_1, τ_2　$\tau=\tau_2-\tau_1$

$$f_o(t)=kf(t-\tau_1)+kf(t-\tau_2)$$

$$F_o(\omega)=kF(\omega)[e^{-j\omega\tau_1}+e^{-j\omega(\tau_1+\tau)}]$$ → 多径效应.

$$H(\omega)=\frac{F_o(\omega)}{F(\omega)}=\underline{ke^{-j\omega\tau_1}}\boxed{(1+e^{-j\omega\tau})}$$

$|H'(\omega)|$

$$|H'(\omega)|=|1+e^{-j\omega\tau}|=2\left|\cos\frac{\omega\tau}{2}\right|$$

当 $\omega=\frac{2\pi}{\tau}\cdot n$ 最大

当 $\omega=\frac{(2n+1)\pi}{\tau}$ 0

频率选择性衰落.

Transmission Pole　Attenuation Minimum (极点，最小衰落)

Transmission Null　Attenuation Maximum (零点：最大衰落)

Channel Coherence Bandwidth (信道相干带宽)

$\Delta f=\frac{1}{\tau}$　τ: 时延差　相邻传输零点的频率间隔　一般 $(1/3\sim1/5)\Delta f$

减小频选衰落 即 信道带宽 $B_s<\Delta f$

Nyquist: $C=2B\log_2 M$

符号宽度与时延差的关系: $T_B>\tau$　　$R_B=\frac{1}{T_B}$

T_B (码元周期)

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最小径迫相干带宽

两径 → 多径 $\boxed{\Delta f = 1/\tau_m}$ → maximum Delay

channel capacity: 无差错传输下最大平均信息速率.

1) Nyquist Theorem.

带宽为 B (Hz) 的无扰信道, 其所承载的最大信息速率为

$$C = 2B\log_2 M$$

M: 进制
B: 带宽

2) Shannon: 在AWGN信道中, error-free

$$C = B\log_2\left(1+\frac{S}{N}\right) \quad (b/s)$$

S - 信号平均功率. B - 带宽(Hz) $N = n_0 B$ (n_0: 噪声单边功率谱密度)

$$\begin{cases} S\to\infty, & C\to\infty \\ n_0\to 0, & C\to\infty \\ B\to\infty, & C\to 1.44\frac{S}{n_0} \text{ (信道容量极限)} \end{cases}$$

Discrete:

互信息 Mutual Information:

$$I(x_i, y_j) = I(x_i) - I(x_i|y_j) = \log_2\frac{1}{p(x_i)} - \log_2\frac{1}{p(x_i|y_j)} \quad \text{(single symbol)}$$

$$I(X;Y) = E\{I(x_i)\} - E\{I(x_i|y_j)\}$$

$p(x_i y_j) = p(x_i|y_j)\cdot p(y_j)$

$$= \sum_{i=1}^{n} p(x_i)\log_2\frac{1}{p(x_i)} - \sum_{j=1}^{m}\sum_{i=1}^{n} \underline{p(x_i y_j)}\log_2\frac{1}{p(x_i|y_j)}$$

$$\sum_{j=1}^{m} p(y_j)\boxed{\sum_{i=1}^{n} p(x_i|y_j)\log_2\frac{1}{p(x_i|y_j)}} = H(X|y_j)$$

$$= H(X) - H(X|Y)$$

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最大信息量 即为 信道容量 (Discrete)

每个符号: $C = \max[H(X) - H(X|Y)]$ bit/symbol

每秒钟: $C_t = \max\{R_B[H(X) - H(X|Y)]\}$ bit/s

无噪声: $H(X|Y) = 0$ 噪声很大: $H(X) = H(X|Y)$

$H(X|Y)$: 因噪声而损失而平均信息量

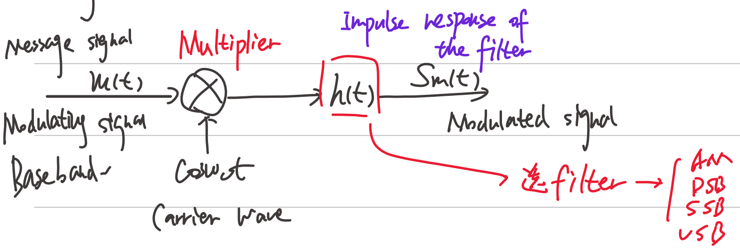
Continuous-Wave Modulation: 连续波调制

1. Carrier(载波) 正弦波
2. 分类: 线性(Linear): AM. DSB. SSB. VSB
   非线性(Nonlinear): FM. PM
3. 目的: ① 减小天线尺寸 ② 频谱搬移(复用) ③ 扩展带宽(抗干扰)

Linear Modulation Schemes (线性调制)

Modulation theorem (调制定理):

$$f(t)\cos\omega_c t \Leftrightarrow \frac{1}{2}[F(\omega-\omega_c)+F(\omega+\omega_c)]$$



$S_m(t) = [m(t)\cos W_c t] * h(t)$

$S_m(w) = \frac{1}{2}[M(w+W_c) + M(w-W_c)]\cdot H(w)$

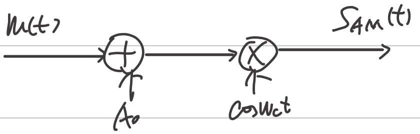
AM: (幅度调制)　　A_0: 直流偏置 (DC bias)

$S_{AM}(t) = [A_0 + m(t)]\cos W_c t$

$= A_0\cos W_c t + m(t)\cos W_c t$

Carrier component 载波分量　　sideband component 边带分量

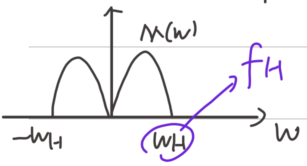
AM Modulator:

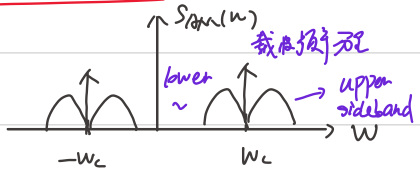


标准AM波: $s(t) = A_c[1+k_a m(t)]\cos W_c t$　调幅灵敏度.

$S_{AM}(w) = \pi A_0[\delta(W+W_c)+\delta(w-W_c)] + \frac{1}{2}[M(w+W_c) + M(w-W_c)]$

包络检波: 条件: $\boxed{|m(t)|_{max} \le A_0}$　通过包络提取信号m(t)





AM的传输带宽是调制信号的两倍　$B_{AM} = 2f_H$　调制信号最高频率

Modulation efficiency.

$P_{AM} = \overline{S^2_{AM}(t)} = \overline{[A_0+m(t)]^2\cos^2 W_c t}$　　$\overline{m(t)} = 0$

$= \overline{A_0^2\cos^2 w_c t} + \overline{m^2(t)\cos^2 w_c t} + \boxed{\overline{2A_0 m(t)\cos^2 W_c t}} = 0$

$= \frac{A_0^2}{2} + \frac{\overline{m^2(t)}}{2} = P_c + P_s$

$\eta_{AM} = \frac{P_s}{P_{AM}} = \frac{\overline{m^2(t)}}{A_0^2 + \overline{m^2(t)}}$

调制效率

∵ $|m(t)| \le A_0$

∴ $\eta_{AM} < 50\%$

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Modulation Index

调幅系数 $m=\frac{|m(t)|_{max}}{A_0}$

如果 $m(t)=A_m\cos\omega_m t$ (单音) $\overline{m^2(t)}=A_m^2/2$

$$\eta_{AM}=\frac{A_m^2}{2A_0^2+A_m^2}=\frac{m^2}{2+m^2}$$

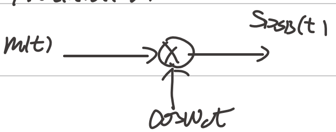
当 $m=1$ 时 η_{AM} 最大即为 $1/3$

DSB(双边带调制): 在AM基础上去掉直流偏置A_0.

时域表达式: $S_{DSB}(t)=m(t)\cos\omega_c(t)$

$$S_{DSB}(\omega)=\frac{1}{2}[M(\omega+\omega_c)+M(\omega-\omega_c)]$$

Modulator:

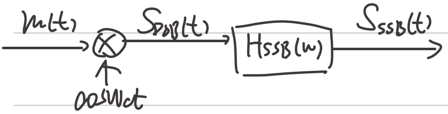


$\eta_{DSB}=100\%$ $B_{DSB}=B_{AM}=2f_H$ (不可用包络检波, 可用相干解调

SSB(单边带调制)

steps: ① 产生一个DSB ② filter out one sideband

1) 滤波法 Filtering Method



USB(保留上边带): $H(\omega)=H_{USB}(\omega)=\begin{cases}1 & |\omega|>\omega_c\\ 0 & |\omega|\le\omega_c\end{cases}$

LSB(保留下边带): $H(\omega)=H_{LSB}(\omega)=\begin{cases}1 & |\omega|\le\omega_c\\ 0 & |\omega|>\omega_c\end{cases}$

边带滤波器难以制作

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$S_{SSB}(\omega) = S_{DSB} \cdot H(\omega)$

2) 相移法: Phase-Shifting Method

以单频调制为例:

$m(t) = A_m \cos \omega_m t \qquad c(t) = \cos \omega_c t$

$S_{DSB} = A_m \cos \omega_m t \cos \omega_c t$

$= \frac{1}{2} A_m \cos(\omega_c + \omega_m)t + \frac{1}{2} A_m \cos(\omega_c - \omega_m)t$

USB　　　　LSB

$S_{LSB}(t) = \frac{1}{2} A_m \cos(\omega_c - \omega_m)t$

USB　　　　+

$S_{LSB}(t) = \frac{1}{2} A_m \cos \omega_c t \cos \omega_m t + \frac{1}{2} A_m \sin \omega_c t \boxed{\sin \omega_m t}$

$S_{USB}(t) = \frac{1}{2} A_m \cos \omega_c t \cos \omega_m t - \frac{1}{2} A_m \sin \omega_c t \boxed{\sin \omega_m t}$

$\sin \omega_m t$ 由 $\cos \omega_m t$ 相移 $\pi/2$ 此过程即为 Hibert transform.

$A_m \widehat{\cos} \omega_m t = A_m \sin \omega_m t$

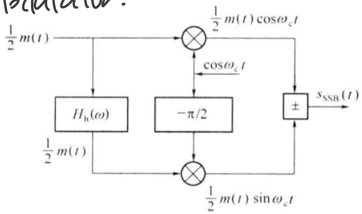
$\Rightarrow S_{SSB}(t) = \frac{1}{2} A_m \cos \omega_m t \cos \omega_c t \pm \frac{1}{2} A_m \widehat{\cos} \omega_m t \sin \omega_c t$

即 $S_{SSB}(t) = \frac{1}{2} m(t) \cos \omega_c t \pm \frac{1}{2} \hat{m}(t) \sin \omega_c t$　　　+ LSB　− USB

$m(t) \longleftrightarrow M(\omega) \qquad \hat{m}(t) \longleftrightarrow M(\omega) \cdot [-j \,\mathrm{sgn}\, \omega]$

$H_h(\omega) = \frac{\hat{M}(\omega)}{M(\omega)} = -j \,\mathrm{sgn}\, \omega$　　　sgn: 符号函数.

Modulator:

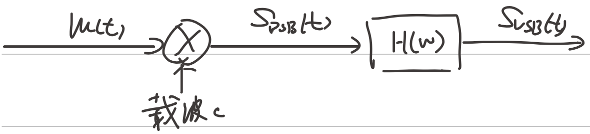


difficulty: (1)必须对 $m(t)$ 所有频分量精确相移 $\frac{\pi}{2}$

$B_{SSB} = f_H$　频带利用率高．　也只能相干解调．

3) VSB (残留边带调制)

Modulator:



$$S_{VSB}(\omega) = S_{DSB}(\omega)\cdot H(\omega) = \frac{1}{2}[M(\omega+\omega_c)+M(\omega-\omega_c)]H(\omega)$$

从接收端去分析．确定 VSB 的 filter 满足的性质

$$S_p(t) = 2S_{VSB}(t)\cos\omega_c t$$

代入

$$S_p(\omega) = [S_{VSB}(\omega+\omega_c) + S_{VSB}(\omega-\omega_c)]$$

$$S_p(\omega) = \frac{1}{2}[M(\omega+2\omega_c)+M(\omega)]H(\omega+\omega_c) + \frac{1}{2}[M(\omega)+M(\omega-2\omega_c)]H(\omega-\omega_c)$$

低通滤去

在 $\pm\omega_c$ 处是互补对称 (奇对称)

= constant

经过 LPF: $S_p(\omega) = \frac{1}{2}M(\omega)\boxed{[H(\omega+\omega_c)+H(\omega-\omega_c)]}$　无失真恢复

$|\omega| \le \omega_H$

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传输带宽: $f_H < B_{SSB} < 2f_H$

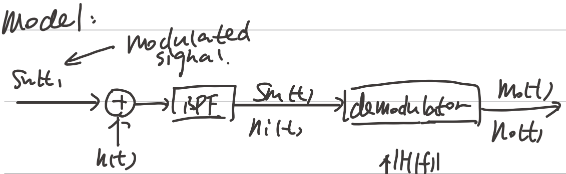
解调方法: 相干解调 (滤波器有互补对称性)

应用: video signal

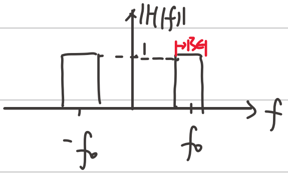
线性调制系统的抗噪声性能

Anti-noise performance of linear demodulation

model:



BPF: 带通滤波器



DSB信号: $B=2f_H$ $f_0=f_c$

SSB信号: $B=f_H$ $f_0=f_c\pm\frac{1}{2}f_H$

$n_i(t) \rightarrow$ Narrowband Gaussian Noise

$n_i(t)=n_c(t)\cos w_0t-n_s(t)\sin w_0t$ (同相 正交)

$\overline{n_i^2(t)}=\overline{n_c^2(t)}=\overline{n_s^2(t)}=N_i=n_0B$

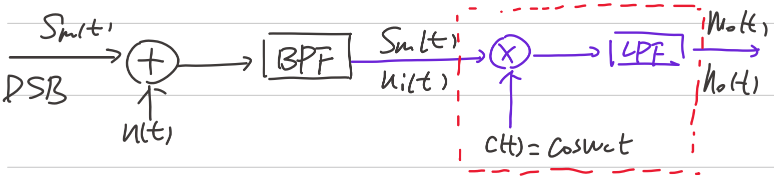
输出信噪比: $\frac{S_o}{N_o}=\frac{\text{有用信号}}{\text{噪声}}=\frac{\overline{m_o^2(t)}}{\overline{n_o^2(t)}}$

不同解调器: 信噪比增益 $G=\frac{S_o/N_o}{S_i/N_i}=\frac{\overline{s_m^2(t)}}{\overline{n_i^2(t)}}$ 调制系统增益

1) DSB performance

相干解调

Coherent Demodulator



$S_i = \overline{S_m^2(t)} = \overline{[m(t)\cos w_c t]^2} = \frac{1}{2}\overline{m^2(t)}$

输入: $S_m(t) = m(t)\cos w_c t$ $\qquad$ $c(t) = \cos w_c t$

$\therefore m(t)\cos^2 w_c t = \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos 2w_c t$

$\xrightarrow{LPF} m_o(t) = \frac{1}{2}m(t)$

$S_o = \overline{m_o^2(t)} = \frac{1}{4}\overline{m^2(t)}$

噪声部分:

$c(t)\,n_i(t) = n_c(t)(\cos w_c t)^2 - n_s(t)\sin w_c t\cos w_c t$ → 正交分量. Quadrature Component

$= \frac{1}{2}n_c(t) + \frac{1}{2}[n_c(t)\cos 2w_c t - n_s(t)\sin 2w_c t]$

$\xrightarrow{LPF} n_o(t) = \frac{1}{2}n_c(t)$ $\qquad$ $N_o = \overline{n_o^2(t)} = \frac{1}{4}\overline{n_c^2(t)} = \frac{1}{4}\overline{n_i^2(t)} = \frac{1}{4}N_i = \frac{1}{4}n_oB$

输入信噪比: $\frac{S_i}{N_i} = \frac{\frac{1}{2}\overline{m^2(t)}}{n_oB}$ $\qquad$ $\boxed{B = 2f_H}$

输出: $\frac{S_o}{N_o} = \frac{\frac{1}{4}\overline{m^2(t)}}{\frac{1}{4}n_oB} = \frac{\overline{m^2(t)}}{n_oB}$

$G = \frac{S_o/N_o}{S_i/N_i} = 2$ $\qquad$ 因为相干解调使一个正交分量 $n_s(t)$ 被消除了

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2) SSB performance (model 同 DSB)

$N_o = \frac{1}{4} N_i = \frac{1}{4} n_o B$ $\boxed{B = f_H}$

$S_m(t) = \frac{1}{2} m(t) \cos w_c t \mp \boxed{\frac{1}{2} \hat{m}(t) \sin w_c t}$ 正交分量.

$\xrightarrow[LPF]{c(t)} m_o(t) = \frac{1}{4} m(t)$

$S_o = \overline{m_o^2(t)} = \frac{1}{16} \overline{m^2(t)}$

$S_i = \overline{S_m^2(t)} = \frac{1}{4} [\frac{1}{2} \overline{m^2(t)} + \frac{1}{2} \overline{\hat{m}^2(t)}]$

$= \frac{1}{4} \overline{m^2(t)}$

输入信噪比: $\frac{S_i}{N_i} = \frac{\frac{1}{4} \overline{m^2(t)}}{n_o B}$

输出〃: $\frac{S_o}{N_o} = \frac{\frac{1}{16} \overline{m^2(t)}}{\frac{1}{4} n_o B} = \frac{\overline{m^2(t)}}{4 n_o B}$

$G = \frac{S_o/N_o}{S_i/N_i} = 1$ (信号与噪声的正交分量均被抑制, 所以SNR无改善)

在SSB中: $\frac{S_o}{N_o} = \frac{S_i}{N_i} \to B = f_H$

在DSB中: $\frac{S_o}{N_o} = 2 \cdot \frac{S_i}{N_i} \to 2 f_H$

$\therefore \left(\frac{S_o}{N_o}\right)_{DSB} = \left(\frac{S_o}{N_o}\right)_{SSB}$

两者抗噪声性能是相同的. 但SSB带宽仅为DSB的一半.

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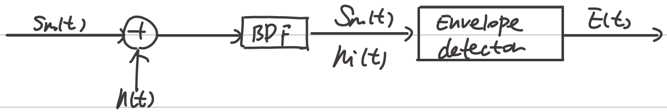
AM包络检波的性能.

Model:



$S_m(t)=[A_0+m(t)]\cos\omega_c t$ $\qquad S_i=\overline{s_m^2(t)}=\frac{A_0^2}{2}+\frac{\overline{m^2(t)}}{2}$

$n_i(t)=n_c(t)\cos\omega_c t-n_s(t)\sin\omega_c t$ $\qquad N_i=n_0B$

input: $S_m(t)+n_i(t)=[A_0+m(t)+n_c(t)]\cos\omega_c t-n_s(t)\sin\omega_c t$

$$=E(t)\cos[\omega_c t+\varphi(t)]$$

$E(t)=\sqrt{[A_0+m(t)+n_c(t)]^2+n_s^2(t)}$ $\qquad \varphi(t)=\arctan\left[\frac{n_s(t)}{A_0+m(t)+n_c(t)}\right]$

① Large SNR: $[A_0+m(t)]\gg\sqrt{n_c^2(t)+n_s^2(t)}$

$$E(t)\approx A_0+\underbrace{m(t)}_{\text{signal}}+\underbrace{n_c(t)}_{\text{noise}}\xrightarrow[\text{阻隔直流}A_0]{\text{电容器}}$$

$S_o=\overline{m^2(t)}$ $\qquad N_o=n_0B$ $\qquad \therefore G_{AM}=\frac{S_o/N_o}{S_i/N_i}=\frac{2\overline{m^2(t)}}{\overline{m^2(t)}+A_0^2}$

$S_i=\frac{1}{2}[A_0+m(t)]^2$ $\qquad N_i=n_0B$

$=\frac{1}{2}[A_0^2+m^2(t)]$

又$\because$ $|m(t)|_{max}\le A_0 \Rightarrow G_{AM}\le 1$

$A_0^2\downarrow$ $G_{AM}\uparrow$ 单音信号 $G_{AM}=\frac{2}{3}$

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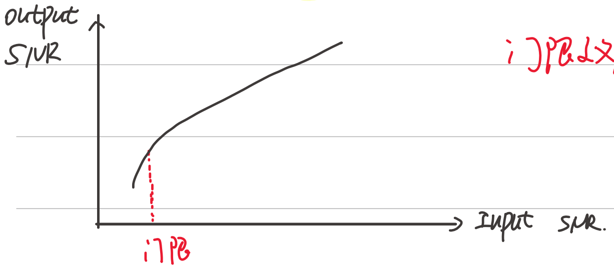
② small SNR. $[A_0+m(t)] \ll \sqrt{n_s^2(t)+n_c^2(t)}$

Let $R(t)=\sqrt{n_c^2(t)+n_s^2(t)}$ $\quad \cos\theta(t)=\dfrac{n_c(t)}{R(t)}$

$E(t)=R(t)+[A+m(t)]\cos\theta(t)$



门限效应 (threshold effect)

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Angle Modulation (角度调制) — 非线性调制

$S_m(t) = A\cos[\omega_c t + \varphi(t)]$ ↪ $m(t)$

carrier wave (constant)

PM 相位偏移: $\varphi(t) = k_p m(t)$

PM sensitivity

k_p: 相位常数 (rad/V)

FM 频率偏移: $\frac{d\varphi(t)}{dt} = k_f m(t)$

k_f: 频偏常数 rad/(s·V)

$S_{PM} = A\cos[\omega_c t + k_p m(t)]$

$S_{FM} = A\cos[\omega_c t + k_f \int m(\tau)d\tau]$

$m(t)$ unknown ⟶ can't determine PM or FM

单音信号调制: $m(t) = A_m\cos\omega_m t = A_m\cos 2\pi f_m t$

1) PM: $\varphi(t) = k_p m(t) = k_p A_m\cos\omega_m t = k_p A_m\cos\omega_m t = m_p\cos\omega_m t$

最大相位偏移 ~ offset

$m_p = k_p A_m$ — 调相指数

$S_{PM}(t) = A\cos[\omega_c t + m_p\cos\omega_m t]$

2) FM: $\frac{d\varphi(t)}{dt} = \boxed{k_f A_m}\cos\omega_m t$ 最大角频偏

$\therefore \varphi(t) = k_f A_m\int\cos\omega_m t\,dt = \frac{k_f A_m}{\omega_m}\sin\omega_m t = m_f\sin\omega_m t$

$m_f = \frac{k_f A_m}{\omega_m} = \frac{\Delta\omega}{\omega_m} = \frac{\Delta f}{f_m}$ 调频指数

最大频偏: $\Delta f = m_f f_m = \frac{\Delta\omega}{2\pi}$ $\Delta\omega = k_f A_m$ $\omega_m = 2\pi f_m$

$S_{FM}(t) = A\cos[\omega_c t + m_f\sin\omega_m t]$

Narrowband FM

$$S_{FM}(t) = A\cos[w_c t + k_f \int m(\tau)\,d\tau]$$

If satisfied: $[k_f \int m(\tau)\,d\tau]_{max} \ll \frac{\pi}{6}$ NBFM.

$$S_{NBFM}(t) = A\cos w_c t \cos[\underbrace{k_f \int m(\tau)\,d\tau}_{\ll \frac{\pi}{6}}] - A\sin w_c t \sin[\underbrace{k_f \int m(\tau)\,d\tau}_{\ll \frac{\pi}{6}}]$$

≈ 1 $\qquad \rightarrow k_f \int m(\tau)\,d\tau$

约等于 $S_{NBFM}(t) \approx A\cos w_c t - [A k_f \int m(\tau)\,d\tau]\sin w_c t$

Frequency domain:

$$S_{NBFM}(w) = \underbrace{\pi A[\delta(w+w_c) + \delta(w-w_c)]}_{\text{carrier.}} + \frac{Ak_f}{2}\left[\frac{M(w+w_c)}{w+w_c} - \frac{M(w-w_c)}{w-w_c}\right]$$

AM: $S_{AM}(w) = \pi A[\delta(w+w_c) + \delta(w-w_c)] + \frac{1}{2}[M(w+w_c) + M(w-w_c)]$

Wideband FM

single-tone: $m(t) = A_m \cos w_m t$

$$S_{FM}(t) = A\cos[w_c t + m_f \sin w_m t].$$

$$= A\cos w_c t \cdot \cos(m_f \sin w_m t) - A\sin w_c t \sin(m_f \sin w_m t)$$

贝塞尔函数 $\quad = A\sum_{n=-\infty}^{\infty} J_n(m_f)\cos(w_c + n w_m)t$

Frequency domain: $S_{FM}(w) = \pi A \sum_{n=-\infty}^{\infty} J_n(m_f)[\delta(w - w_c - n w_m) + \delta(w + w_c + n w_m)]$

带宽: 理论上是无限宽, 但 $J_n(m_f)$ 随n增大而减小. 卡森(Carson)公式

要求: $|J_n(m_f)| \geq 0.1$ $\quad$ $\boxed{\text{有效带宽 } B_{FM} = 2(m_f + 1) f_m = 2(\Delta f + f_m)}$

当 $m_f \ll 1$ 时 $B_{FM} \approx 2 f_m$ — NBFM $\quad$ 当 $m_f \gg 1$ 时 $B_{FM} \approx 2\Delta f$ — WBFM

日期: /

多音信号:

$$B_{FM} = 2(m_f + 1)f_m = 2(\Delta f + f_m)$$

调制信号中的最高频率

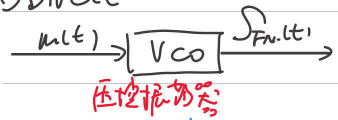
$$S_{FM}(t) = A\sum_{n=-\infty}^{\infty} J_n(m_f)\cos(\omega_c + n\omega_m)t$$

$$P_{FM} = \overline{S_{FM}^2(t)} = \frac{A^2}{2}\underbrace{\boxed{\sum_{n=-\infty}^{\infty} \overline{J_n^2(m_f)}}}_{=1} = \frac{A^2}{2}$$

结论：功率调制前后总功率不变（功率重新分配）

FM signal Generation

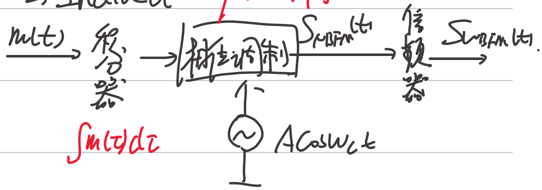
1) Direct



$\omega_i(t) = \omega_0 + k_f m(t)$

2) Indirect

$\phi(t) = k_p\int m(\tau)d\tau$



解调（demodulation）

1) Non-coherent ~ 鉴频（非相干解调）

适用于 NBFM 和 WBFM

discriminator（鉴频器）

（频率-电压转换）

2) Coherent ~ 只适用于 NBFM（需要精确的载波同步）

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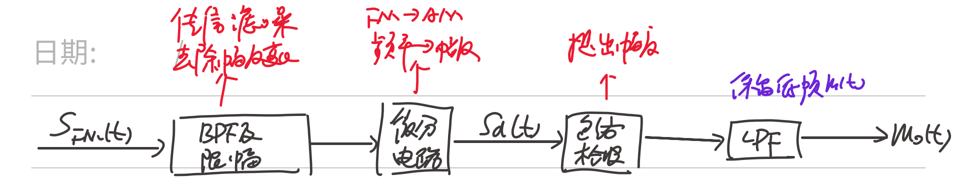


$$S_{FM}(t) = A\cos[w_c t + k_f \int m(\tau) d\tau]$$

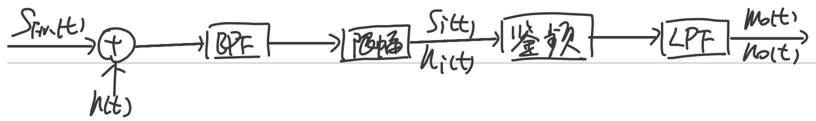
$$S_d(t) = \frac{d}{dt} S_{FM}(t)$$

$$= -A[w_c + k_f m(t)] \sin(w_c t + k_f \int m(\tau) d\tau)$$

包络检波 振幅 $\propto [w_c + k_f m(t)] \xrightarrow[\text{去直流}]{LPF} m_o(t) = K_d k_f m(t)$ 鉴频器的灵敏度

Anti-Noise performance of FM

model:



High SNR: $G_{FM} = 3m_f^2(m_f+1)$

$B_{FM} = 2(m_f+1)f_m = 2(\Delta f + f_m)$

单音信号

抗干扰能力强

频带利用率低

Low SNR: Threshold Effect

Frequency-Division Multiplexing 频分复用

Modulation: frequency shifting (频谱搬移)

目的: ①共享物理信道. 提高信道利用率. ②原理: 仿造我们的互不重叠的通道

①多路载波电话系统 (SSB FDM)

日期:　　/

Pulse Modulation 脉冲调制

three steps: Sampling → quantization → coding

Analog signal $m(t)$ → Sampling signal (时间离散) → Quantized signal (幅值离散) → Encoded / Digital signal (时间, 幅值均离散)

Sampling of low-pass analog signal

T_s : Sampling period 采样时间

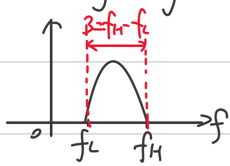
$f_s = 1/T_s$. Sampling Rate 采样率

Sampling Theorem: 最高: $T_s = \frac{1}{2f_H}$ - Nyquist 间隔

最低 $f_s = 2f_H$ - Nyquist 采样速率

Sampling of band-pass analog signal

$[f_L, f_H]$　$B = f_H - f_L$



$f_L < B$ → low-pass

$f_L > B$ → band-pass

Sampling Theorem: if $f_H = nB + kB$.

最小 $f_s = 2B(1 + \frac{k}{n})$　n 是整数 $k \in (0,1)$ k 是小数部分

Carrier: 周期矩形脉冲序列

1. 高度/幅值　A
2. 宽度/持续时间　τ
3. 位置　t_p

调制: 让某一参数随 $m(t)$ 改变

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① PAM：脉冲幅度调制：脉冲的高度在变化

优点：实现简单．缺点：对噪声敏感

② PDM. PWM：脉冲宽度调制：脉冲宽度在变化

③ PPM：脉冲位置调制：脉冲的位置在变化

Natural Sampling：自然抽样．Flat-top Sampling 平顶抽样．

Ideal：$M_s(f)=\frac{1}{T_s}\sum_{n=-\infty}^{\infty} M(f-nf_s)$

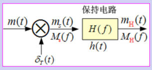
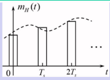
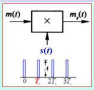
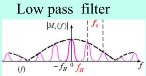
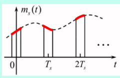
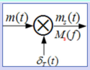
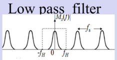
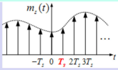
Natural：$M_s(f)=\frac{A\tau}{T_s} Sa(\pi\tau n f_s) M(f-nf_s)$ 被 Sa(·) 加权．

Flat-top. $m(t) \xrightarrow{\times\delta_{T_s}(t)} m_s(t) \xrightarrow{\text{保持电路}h(t)} m_H(t)$

$$M_H(f)=\frac{1}{T_s}H(f)\sum_n M(f-nf_s) \xrightarrow{LPF} \frac{1}{T_s}H(f)M(f)$$

需要再加一个filter的均衡器 ($1/H(f)$) $\longrightarrow \frac{1}{T_s}M(f)$

Flat-top VS Natural sampling

Sampling Mode	generation	recovery	waveform
Flat-top		Low pass filter + **equalizer 1/H(*f*)**	
Natural		Low pass filter 	
Sampling		Low pass filter 	

Quantization 量化.

1. Uniform Quantization (均匀量化) — M intervals.

number of quantization levels (量化电平数) M

quantization interval (量化间隔) $\Delta V = (b-a)/M$

boundary points (端点) $m_i = a + i\Delta V$

信号量噪比 S/N_q : performance indicator

$$\frac{S}{N_q} = \frac{E[m_k^2]}{E[(m_k - m_q)^2]}$$

$$\boxed{N_q = \frac{(\Delta V)^2}{12}} \qquad \Delta V = \frac{2a}{M}$$

input signal: PSD: $[-a, a]$ uniform

$$S = \frac{(\Delta V)^2}{12} \cdot M^2$$

for a given quantizer, N_q 为常数.

$\therefore \frac{S}{N_q} = M^2$ 假设二进制编码位数为 N. $M = 2^N$ $\left(\frac{S}{N_q}\right)_{dB} \approx 6N$

① SNR随 M 的增大而增大 ② 若 S 较小则 $\frac{S}{N_q}$ 变小 → Non uniform

Nonuniform 非均匀量化:

思路: Small signal sample, small ΔV.

方法: x ⟶ 压缩 ⟶ 均匀量化 ·· ⟶ 扩张 ⟶

Compressor　Uniform quantization

一般: log( )

A-Law — 13 segments (China, European)

μ-Law — 15 segments (北美. Japan. Korea).

A-Law: $A=87.6$ → 原点附近斜率为16. → 2的幂次递减 / $A=1$ 无压缩

$$y=\begin{cases}\dfrac{Ax}{1+\ln A} & 0\le x\le \dfrac{1}{A}\\[2ex] \dfrac{1+\ln Ax}{1+\ln A} & \dfrac{1}{A}\le x\le 1\end{cases}\qquad \frac{dy}{dx}=\frac{A}{1+\ln A}=16 \Rightarrow A=87.6$$

将 y均匀分为8份. x不均匀分为8份 (双极性. 正负对称)

$1/8,\ 2/8,\ 3/8 \cdots 1$　　$1/128,\ 1/64 \cdots 1/2$

2^n-1: $\frac{1}{255},\ \frac{3}{255},\ \frac{7}{255} \cdots \frac{127}{255}$

μ-law: $y=\dfrac{\ln(1+\mu x)}{\ln(1+\mu)}$　$0\le x<1$　通常: $\mu=255$. $\mu=0$时 无压缩

总结: 小信号的量噪比是A律的2倍. 大信号量噪比略比A律稍差

Pulse-Code Modulation　PCM　脉冲编码调制

Analog signal input → Sampling + quantizing + encoding → PCM signal

sender

PCM signal → decoding → LBP → Analog signal

Receiver

8 bits in PCM (A-law)

$C_1\ C_2\ C_3\ C_4\ C_5\ C_6\ C_7\ C_8$

C_1 极性码 — the polarity bit $\begin{cases} + & C_1=1 \\ - & C_1=0. \end{cases}$

$C_2C_3C_4$ 段落码 — segment bits. 000~111

$C_5C_6C_7C_8$ 段内码 — inner-segment bits　0000~1111

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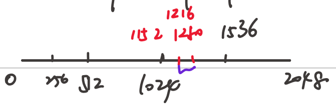
例: +1270. find the output of PCM encoder and the quantization error: +1270



2^n $[1216, 1280)$

$C_1=1$ $C_2C_3C_4=111$ $C_5C_6C_7C_8=0011$ $1270-1216=54\Delta$

⇒ PCM: 11110011 Encoding error: $(I_s - I_c) = 54\Delta$

在译码器中都有一个加 $\Delta_i/2$ 的电路

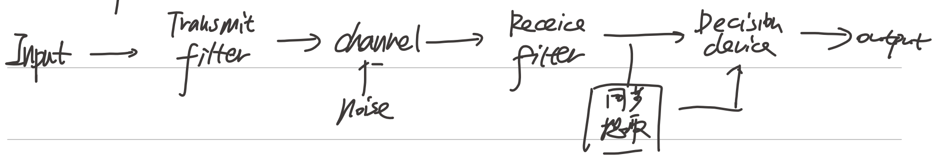
PCM译码不输出左端点,而是输出区域的中点 $\Delta_i = 64\Delta$

$I_D = I_c + \frac{1}{2}\Delta_i = 1216+32 = 1248$ ∴ Decoding Error: $(I_s - I_D) = 22\Delta$

→ 未经调制,占据低频

Baseband pulse transmission 数字基带传输.

Baseband pulse



码型变换、波形变换 / 如何消除ISI / 如何减小信道噪声

线路编码:表示0和1.

Unipolar: "+A" 和 "0"

Bipolar: "+A" 和 "−A"

Unipolar NRZ, Bipolar NRZ

Unipolar RZ, Bipolar RZ

RZ: 归零: $\tau/T_B < 1$ 通常占空比50%

Differential encoding 差分编码 → 用相邻码元电平的变化表示/不变来表示一个信息码元

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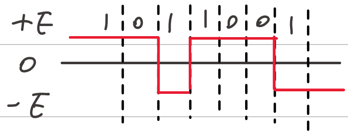
传号差分 (1变 0不变)
空号差分 (0变 1不变)

例:



以上都是 one pulse carries one bit

多电平波形 (Multilevel encoding): one pulse carries several bits

M-ary 码元 ~ k 比特组

四电平:
11 — +3E
10 — +E
00 — -E
01 — -3E

$R_b = R_B \log_2 M$

传输码:

1. AMI码 — 传号交替反转码 Alternative Mark Inversion

Rule: "1" (传号) — +1, -1 alternative.
"0" (空号) — 0

ex. code 1 0 0 1 1 0 0 0 0 0 0 0 1 1 0 1 1

AMI : -1 0 0 +1 -1 0 0 0 0 0 0 0 +1 -1 0 +1 -1

characteristic:

No DC component, Easy to find errors, simple encoding & decoding circuitry.

Weakness.

a long string of zeros ⟶ difficult to obtain the timing information.

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高密度双极性码 3阶

4. HDB3 code — the 3th order High Density Bipolar code.

① 0的个数 ≤3: 按照AMI发送

当连续出现4个0时, 把第4个0换成脉冲, 做特殊跳变.

V (Violation pulse: 违例脉冲/破坏脉冲)

极性规定: 要与前一个非零脉冲同号

B (Balancing pulse 调节脉冲)

替换规则: ① 000V ② B00V (相邻的V的极性必须交替)

000+V ⟷ -B00-V

000-V ⟷ +B00+V

ex. Code 1000 1001 0000 1 0000 11 0000 11 因为相邻的V要

HDB3 Code -1000 +100-1 000-V +1 000+V -1 +1 ~~000+V~~ -B00-V

Decoding.

先把V改成1. -1000 +100-1 000-1 +1 000+1 -1 +1 000+1 11

看见极性相同的1. 1000 100 10000 10000 1 10000 11

改成前面一个加0

其余的+1/-1改为1

characteristic: "0" limited to no more than 3 → easy to extract timing signals

complex coding, simple decoding. error ability

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3. Manchester code: 双相码　Application: LAN

Rule "0"—01 "1"—10

4. CMI code — 传号反转

Rule: "1" — 11, 00 交错出现

"0" — 01

Matched filter: 匹配滤波器

输出信噪比在某一特定时刻达到最大值.

The output SNR reaches its maximum at a certain specific moment

数字信号接收: 关键是抽样判决是否正确.

只考虑 noise, 不考虑 ISI

t_0: 抽样时刻, 通常取 $t_0=T$

k: 缩放系数

时域 $h_{opt}(t)=ks(t_0-t)$

频域 $H_{opt}(f)=kS(f)\cdot e^{-j2\pi fT}$

性质: 最大输出SNR $=\frac{2E}{n_0}$ (E: 脉冲能量. $P_n(f)=\frac{n_0}{2}$ 双边)

噪息清零. 在 T_B 之后置加

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匹配滤波法 $\overset{\text{等价}}{\Longleftrightarrow}$ 相关接收法

发"0": $S_0(t)$ $\quad r(t)=S_i(t)+n(t)$ $\quad h_0(t)=S_0(T_B-t)$ $\quad y_0(t)=r(t)*h_0(t)$ $\quad t=T_B$时进行抽样判决

发"1": $S_1(t)$ $\quad h_1(t)=S_1(T_B-t)$ $\quad y_1(t)=r(t)*h_1(t)$

相关接收法: $z_0=\int_0^{T_B} r(u)S_0(u)\,du$, $\quad z_1=\int_0^{T_B} r(u)S_1(u)\,du$ 比较 z_0 和 z_1

ISI Inter-Symbol Interference

Reason: 系统传输总特性不够理想 $\quad H(\omega)=G_T(\omega)\cdot C(\omega)G_R(\omega)$

① channel bandwidth is limited ② receiving/transmitting filter is poorly designed.

$$\{a_n\}\xrightarrow{d(t)} \boxed{G_T(\omega)} \to \boxed{C(\omega)} \to \oplus \to \boxed{G_R(\omega)} \xrightarrow{r(t)} \boxed{\text{抽样判决器}} \to \{a_n'\}$$

$\oplus$ 输入: $n(t)$

$d(t)=\sum_{n=-\infty}^{\infty} a_n\delta(t-nT_B)$ $\quad y(t)=d(t)*h(t)+n_R(t)$

$y(t)=\sum_{n=-\infty}^{\infty} a_n\boxed{h(t-nT_B)}+n_R(t)$ $\quad t=kT_B+t_0$ 代入 $\quad$ 系统的总延迟

$y(kT_B+t_0)=\sum_n a_n h((k-n)T_B+t_0)+n_R(kT_B+t_0)=\boxed{a_k h(t_0)}+\underbrace{\sum_{n\neq k} a_n[h((k-n)T_B+t_0)]}_{ISI}+\underbrace{n_R(kT_B+t_0)}_{noise}$

k-th symbol sampling value

$ISI=\sum_{n\neq k} a_n[h((k-n)T_B+t_0)]$ $\quad$ 令其=0. 前面码元波形在后面抽样时刻为零

Time-domain con. $h(kT_B)=\begin{cases}1 & k=0\\ 0, & k=\text{another int}\end{cases}$ $\quad$ 奈奎斯特第一准则

Frequency-domain con: $\sum_i H\left(\omega+\frac{2\pi i}{T_B}\right)=T_B$ $\quad |\omega|\le\frac{\pi}{T_B}$

$h(t)\Longleftrightarrow H(\omega)=G_T(\omega)C(\omega)G_R(\omega)$

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Impulse response

sinc $H(\omega) = \begin{cases} T_B, & |\omega| \le \frac{\pi}{T_B} \\ 0, & \text{others} \end{cases} \longrightarrow h(t) = Sa(\frac{\pi}{T_B} t)$

$t = \pm kT_B$ 上抽样，即可消除 ISI

Nyquist Bandwidth / Rate

频带利用率

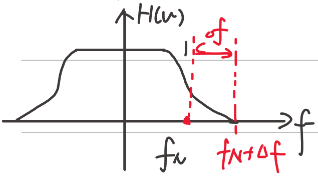
$B = \frac{1}{2T_B} = f_N$ 最窄带宽

$R_B = \frac{1}{T_B} = 2f_N$ 无ISI的最高波特率。

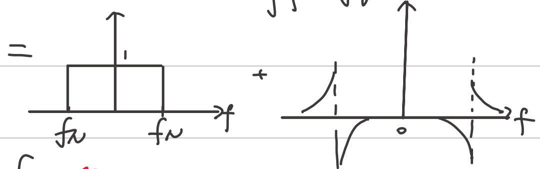
$\eta_B = \frac{R_B}{B} = 2\,(Baud/Hz)$

$\eta_b = \frac{R_b}{B} = 2\cdot\log_2 M$

ideal LPF 难以实现，在 f_N 处按"奇对称"条件进行"滚降"。



Cosine-roll off 特性.



Roll-off Factor: $\alpha = \frac{f_\Delta}{f_N}$ ↑ (0~1)

$(1/t^3)$

$B = f_N + f_\Delta = (1+\alpha) f_N$ ↑

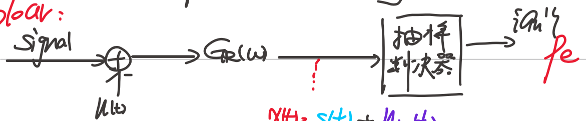
α越大，h(t)拖尾衰减速度越快，有利于减少ISI和位定时误差的影响

$\eta_B = \frac{R_B}{B} = \frac{2}{1+\alpha}$ ↓

$\eta_b = \frac{2}{1+\alpha}\cdot\log_2 M$ ↓

Error rate P_e caused by $n(t)$ when without ISI

bipolar:



$x(t) = s(t) + n_R(t)$

$$\begin{cases} E[n(t)] = 0 \\ P_n(f) = \frac{n_0}{2} \end{cases}$$

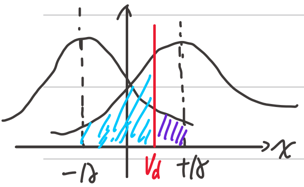
$P_R(f) = \frac{n_0}{2}|G_R(f)|^2$ $\quad n_R \sim N(0, \sigma_n^2)$

$\sigma_n^2 = \int_{-\infty}^{\infty} \frac{n_0}{2}|G_R(f)|^2 df$ $\quad x(t) \sim N(\pm A, \sigma_n^2)$

$$x(kT_B) = \begin{cases} A + n_R(kT_B), & \text{"1"} \\ -A + n_R(kT_B), & \text{"0"} \end{cases}$$

"1": $f_1(x) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left(-\frac{(x-A)^2}{2\sigma_n^2}\right)$ $+A$

"0": $f_0(x) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left(-\frac{(x+A)^2}{2\sigma_n^2}\right)$ $-A$



V_d: threshold

$P(1/0) = P(x > V_d) = \frac{1}{2} - \frac{1}{2}\mathrm{erf}\left(\frac{V_d + A}{\sqrt{2}\sigma_n}\right)$

$P(0/1) = P(x \le V_d) = \frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{V_d - A}{\sqrt{2}\sigma_n}\right)$

$P_e = P(0)\cdot P(1/0) + P(1)\cdot P(0/1)$ 　找到使 P_e 最小的 V_d

令 $\frac{\partial P_e}{\partial V_d} = 0 \Rightarrow V_d^* = \frac{\sigma_n^2}{2A}\ln\frac{P(0)}{P(1)}$ 　当 $P(0) = P(1)$ 时, $\boxed{V_d^* = 0}$

此时 $P_e = \frac{1}{2}[P(0/1) + P(1/0)] = \frac{1}{2}\mathrm{erfc}\left(\frac{A}{\sqrt{2}\sigma_n}\right)$ 　$\frac{A}{\sigma_n}\uparrow$ $P_e\downarrow$

unipolar: 只是把 $-A$ 移到 0 　$V_d^* = \frac{A}{2}$ 　$P_e = \frac{1}{2}\mathrm{erfc}\left(\frac{A}{2\sqrt{2}\sigma_n}\right)$

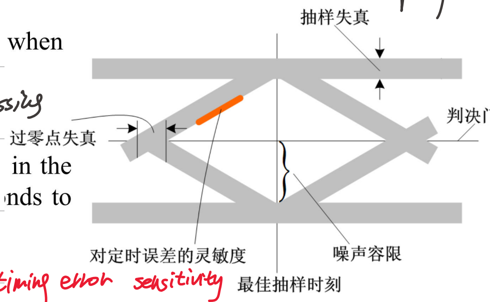
Eye pattern/diagram 眼图.

把多个码元周期的波形叠在同一时间窗口里显示 (设$T_c=T_s$)

主要用于观察: ① ISI ② noise → 看眼线条粗细 ⟨ 粗 有noise / 细 无noise

眼睛张开的大小 ⟨ 大 无ISI / 小 有ISI

Sampling distortion



zero-crossing distortion

threshold

timing error sensitivity

日期:　　　/

Passband Data Transmission 数字带通传输

includes modulation/demodulation processes.

Digital modulation: digital baseband signal $\xrightarrow{\text{Control}}$ parameter/carrier wave.

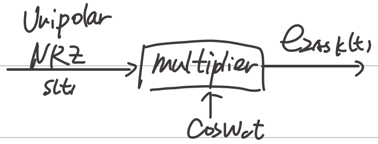
Methods: ① Analog Modulation ② Digital keying

Binary Amplitude Shift keying (2ASK) = 二进制幅移键控

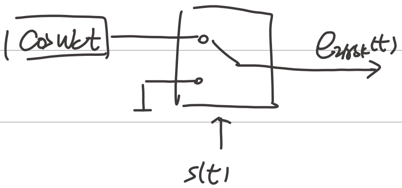
$s(t) \rightarrow$ Carrier Wave Amplitude

Generation:　　　　OOK (on-off keying)

Analog:



Keying:

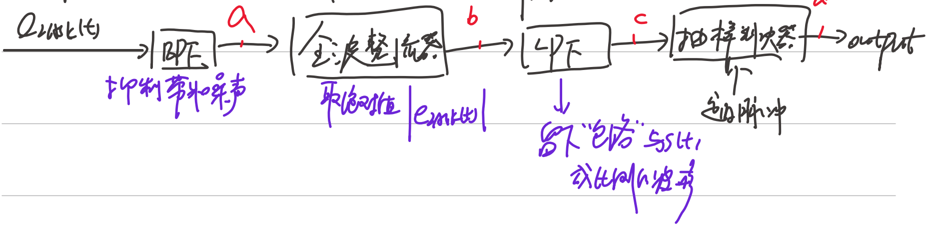


$e_{2ASK}(t) = s(t)\cos w_c t$ ← Unipolar

$s(t) = \sum_n a_n g(t - nT_s)$

Demodulation: $e_{2ASK}(t) = s(t)\cos w_c t$

Envelope Detection Method



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Coherent Demodulation Method:

$e_{2ASK}(t) \rightarrow$ BPF $\rightarrow$ multiplier $\rightarrow$ LPF $\rightarrow$ 抽样判决器 $\rightarrow$ output.

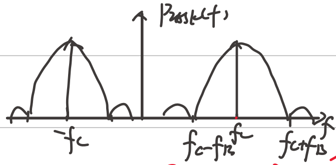
multiplier: $c(t) = \cos\omega_c t$

抽样判决器: 定时脉冲.

$$e_{2ASK}(t)\cdot\cos\omega_c t = s(t)\cdot\cos\omega_c t\cdot\cos\omega_c t = s(t)\cdot\frac{1+\cos 2\omega_c t}{2} \xrightarrow{LPF} \boxed{\frac{1}{2}s(t)}$$

抽样判决

2ASK PSD



unipolar

$e_{2ASK}(t) = s(t)\cdot\cos\omega_c t$

$$P_{2ASK}(f) = \frac{1}{4}[P_s(f+f_c)+P_s(f-f_c)]$$

$B_{2ASK} = 2f_B$ $f_B = \frac{1}{T_B} = R_B$

点: Contains a carrier component (含载波分量)

均值不是0 (有直流分量) → 在 $f=\pm f_c$ 处有载波分量.

Anti-noise: Unipolar system:

$$V_d^* = \frac{A}{2} + \frac{\sigma_n^2}{A}\ln\frac{P(0)}{P(1)} \quad \text{等概} \rightarrow V_d^* = \frac{A}{2} \quad P_e = \frac{1}{2}\mathrm{erfc}\left(\frac{A}{2\sqrt{2}\sigma_n}\right)$$

Binary Frequency-shifting keying (2FSK) 二进制频移键控

$s(t) \xrightarrow{\text{control}}$ Carrier wave freq.

可视为两个不同载频的2ASK的叠加

$$e_{2FSK} = s_1(t)\cos\omega_1 t + s_2(t)\cos\omega_2(t)$$

$$s_1(t) = \sum a_n g(t-nT_s) \quad s_2(t) = \sum \bar{a}_n g(t-nT_s)$$

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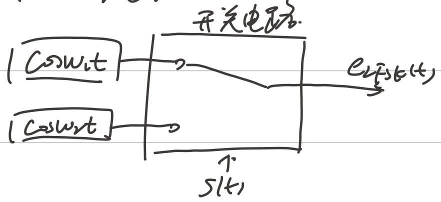
Generation:

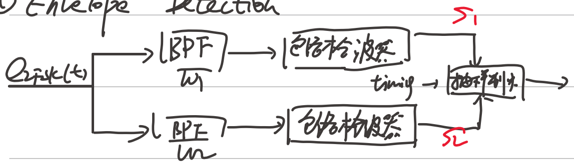
① Analog Freq. Modulation Method

② key shifting

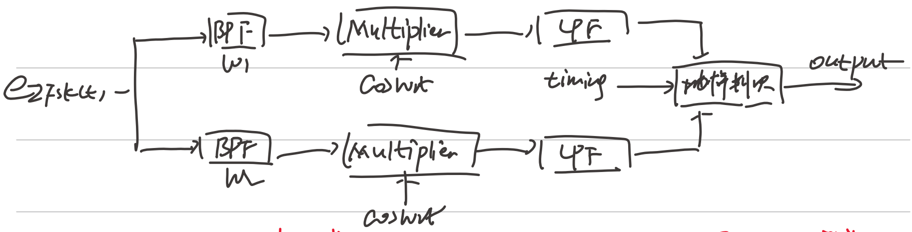


Demodulation: 原理. 将2FSK信号分解为上下两路2ASK信号分别进行解调. 再抽样判决

① Envelope Detection



② Coherent Modulation:



Modulation Rule: ω_1 = "1"
ω_2 = "0"

Judgement Rules: $s_1 > s_2$: "1"
$s_1 \le s_2$: "0"

2FSK PSD

$$e_{2FSK}(t) = s_1(t)\cos\omega_1 t + s_2\cos\omega_2 t$$

$$P_{2FSK}(f) = \frac{1}{4}[P_{s1}(f-f_1) + P_{s1}(f+f_1)] + \frac{1}{4}[P_{s2}(f-f_2) + P_{s2}(f+f_2)]$$

日期: /

$B_{2FSK} \approx |f_2-f_1| + 2f_B$

连续谱形状随着两个载频之差的大小而变化.

$|f_2-f_1| > 2f_B$ 双峰 $|f_2-f_1| < f_B$ 单峰

Binary Phase-shifting keying (2PSK) 二进制相移键控

$e_{2psk}(t) = s(t)\cos\omega_c t$ $s(t) = \sum_n a_n g(t-nT_s)$ $a_n = \begin{cases} 1 & P \\ -1 & 1-P \end{cases}$

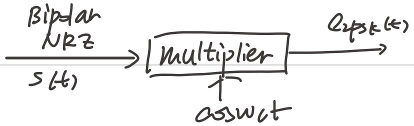
Bipolar

$e_{2psk}(t) = A\cos(\omega_c t + \varphi_n)$ $\varphi_n = \begin{cases} 0 & \text{"0"} \\ \pi & \text{"1"} \end{cases}$

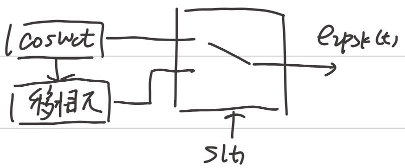
$e_{2psk}(t) = \begin{cases} A\cos\omega_c t \\ -A\cos\omega_c t \end{cases}$

Generation.

① Analog:



② keying:



Demodulation: Only Coherent Demodulation

同上. LPF 输出信号仍为 s(t)

2PSK Existing problem: 180° phase ambiguity 相位模糊.

载波 $\cos w_c t$ (同相)　　经过LPF 除判 - $\frac{1}{2}s(t)$　"1" → "0"　"0" → "1"　整串比特被翻转

$-\cos w_c t = \cos(w_c t + \pi)$ 反向.

solution: 2DPSK

Binary Differential phase shift keying (2DPSK)

原理: 利用前后相邻码元的载波相对相位表示信息

$$\Delta\varphi = \varphi_n - \varphi_{n-1} = \begin{cases} 0 \to \text{"0"} \\ \pi \to \text{"1"} \end{cases}$$

若以相位为载波基准相位 → 绝对调相, 2PSK

前一码元　　相对　　2DPSK

相位不变 → 比特"0"
相位翻转 → 比特"1"

ex.

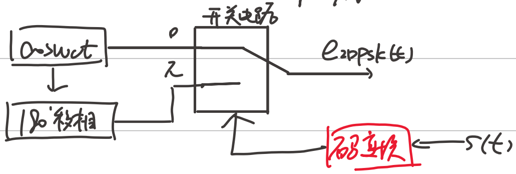
Code	1	1	0	1	0
2DPSK (0)	π	0	0	π	π
信号相位 (π)	0	π	π	0	0

$$\Delta\varphi = \varphi_{n+1} - \varphi_n = \begin{cases} 0 \to \text{"0"} \\ \pi \to \text{"1"} \end{cases}$$

Differential encoding rule: $b_n = a_n \oplus b_{n-1}$

Generation: $\{a_n\}$ (绝对码) → 差分编码 → $\{b_n\}$ 相对码 → 2PSK调制器 → $e_{2DPSK}(t)$

model:



2DPSK Demodulation:　　(Differential coding)

① Coherent Demodulation + Code Inverse Transformation Method

$e_{2DPSK}(t)$ → BPF → multiplier → LPF → 抽样判决器 → 码反变换 → output

定时脉冲 → 抽样判决器

② 差分相干解调法(相位比较法)

Differential Coherent Demodulation (phase comparison)

$e_{2DPSK}(t)$ → BPF → multiplier → LPF → 抽样判决器 → output

BPF → 延迟 T_B → multiplier

定时脉冲 → 抽样判决器

起到了相位比较的作用.

相乘结果反映了前后码元的相位差.

∴不需要差分译码

2PSK PSD:

$$e_{2psk}(t)=s(t)\cos w_c t$$

$$P_{2psk}(f)=\frac{1}{4}[P_s(f+f_c)+P_s(f-f_c)]$$

$P_s(f)$: 双极性随机矩形脉冲序列的功率谱

$P_{2psk}(f)$: No discrete spectrum (无载波分量): 相当于抑制载波的双边带信号)

Anti-noise performance for binary system

For 2ASK-coherent demodulation, the total error of the sys. is:

$$P_e=\frac{1}{2}\mathrm{erfc}\left(\frac{a}{2\sqrt{2}\sigma_n}\right)$$

$\boxed{令\ r=\frac{a^2}{2\sigma_n^2}}$ 解调器输入端信噪比

$a=kA$ (k: channel)

$\sigma_n^2=n_0B=n_0\cdot\frac{2}{T_B}$

当 $r>>1$, $P_e\approx\frac{1}{\sqrt{\pi r}}e^{-r/4}$

For 2ASK-Envelope detection.

下降 → $P_e=\frac{1}{2}e^{-r/4}$

最优门限 $b^*=\frac{a}{2}$ $(r>>1)$

Total error: $P_e=\frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{r}{4}}\right)+\frac{1}{2}e^{-r/4}$

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导论: ① 当 r 相同时，$P_{e\text{相干}} < P_{e\text{包络}}$

② 当 $r \gg 1$ 时，两者性能相差不大

For 2FSK-coherent demodulation:

the total error rate is $P_e = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{r}{2}}\right)$ $r = \frac{a^2}{2\sigma_n^2}$

If $r \gg 1$ $P_e \approx \frac{1}{\sqrt{2\pi r}} e^{-\frac{r}{2}}$

在大信噪比下，二者相差不大。但是 coherent demodulation 设备复杂得多

For 2FSK-envelope demodulation. $P_e = \frac{1}{2} e^{-r/2}$

For 2PSK-coherent demodulation:

$P_e = \frac{1}{2}\operatorname{erfc}\left(\frac{a}{\sqrt{2}\sigma_n}\right)$ 令 $r = \frac{a^2}{2\sigma_n^2} \Rightarrow P_e = \frac{1}{2}\operatorname{erfc}(\sqrt{r})$ $x \gg 1$ 时

$\operatorname{erfc}(x) \sim \frac{e^{-x^2}}{x\sqrt{\pi}}$

当 $r \gg 1$ 时，$P_e \approx \frac{1}{2\sqrt{\pi r}} e^{-r}$.

For 2DPSK-Coherent: 考虑码反变换后影响即可

$P_e' \approx 2P_e \approx \frac{1}{\sqrt{\pi r}} e^{-r}$

For 2DPSK-Differential Coherent demodulation: 差分相干解调

$P_e = \frac{1}{2} e^{-r}$

Conclusion:

1) P_e —— reliability

① 当 r 一定时，相同的解调方式下，抗高斯白噪声性能优劣的顺序如下:

2PSK, 2DPSK, 2FSK. 2ASK / $P_{e\text{相干}} < P_{e\text{非相干}}$

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② P_e一定时，所需信噪比：$r_{2ASK} = 2r_{2FSK} = 4r_{2PSK}$

( )dB = 3dB + ( )dB = 6dB + ( )dB

2) Bandwidth —— effectiveness

$R_B = \frac{1}{T_B}$　　$B_{2ASK} = B_{2PSK} = B_{2DPSK} = 2R_B = \frac{2}{T_B}$

$B_{2FSK} = |f_2 - f_1| + \frac{2}{T_B}$　　R_B一定　2FSK占较带利用率最低

3) Sensitivity to changes in channel characteristics

2ASK：$b^* = a/2$　易受信道参数变化影响，不适用于在变参信道中传输，

2PSK：$b^* = 0$（判决）不易受信道参数影响

2FSK：不需要人为设置判决门限，适用于变参信道传输场合

4) Complexity of the equipment.

Coherent demodulation equipment is more complex / cost is higher.

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M-ary digital modulation

Binary: Each symbol carries 1 bit info.

M-ary: $\log_2 M$

$R_b = R_B \log_2 M$ ① R_b一定 M↑ R_B↓ 带宽减小,节省信道资源

② R_B一定. M↑ R_b↑ η_b↑

代价: ① P_e↑ ② decision region↓; ③ complexity of sys.↑

MASK: extension of 2ASK 多进制幅移键控

$$e_{MASK}(t) = \sum_{n=1}^{M} a_n g(t-nT_s)\cos\omega_c t$$

$$a_n = \begin{cases} 0 & P_1 \\ 1 & P_2 \\ \vdots & \vdots \\ M-1 & P_M \end{cases} \quad \sum_{i=1}^{M} P_i = 1$$

例 4ASK → 振幅有4种取值,每个码元含2bit

MFSK: extension of 2FSK 多进制频移键控

例 4FSK → 用4种不同的频率分别表示双比特信息

$B = |\underline{f_M - f_1}| + 2R_B$ 带宽较大. 频带利用率低. 常用于调制速率不高的衰落信道传输场合

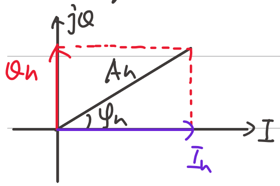
$s_i = A\cos(2\pi f_i t)$

最低频到最高频

M-ary PSK $\theta_n = (n-1)\frac{2\pi}{M}$ $n=1,2,\dots,M$

$$s_n(t) = A\cos\left(2\pi f_c t + \frac{2\pi}{M}(n-1)\right)$$

星座图/信号矢量图 constellation diagram



第n个符号对应的已调信号 → 表示为一个向量

模: 信号幅值　辐角: 信号相位

I: In-phase 同相分量　Q: Quadrature 正交分量

星座图 = 所有可能符号的端点 (I_n, Q_n)

M-PSK: 幅值基本固定,信息只在相位上: $\begin{cases} A_n = A \\ \varphi_n = \frac{2\pi k}{M} \end{cases}$ 半径为A的圆上均匀分布的M个点

Increasing M: ① 相同带宽中传输更多比特信息,从而提高频带利用率(η)

② 相邻信号点距离减少(判决范围减小,噪声容限减小)。 相邻点间的距离

降低 P_e 方法: 用格雷码进行编码: 一个符号错误 → 大概率只引 1 bit 错.

QAM 调制 振幅、相位联合键控的调制方式 (APK调制)

16-QAM. a) 方型 —最常用 b) 星型

Generation: 1路ASK × 1路ASK.

a) 复合相移法(较少使用) b) 2路4ASK正交法(主流)

I路: 4ASK (=4PAM) 取4个幅度电平(2bit)
Q路: 4ASK (=4PAM) 取4个幅度电平(2bit) → 分别用 I.Q → 调制载波

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OFDM 正交频分复用

basic idea: serial → parallel

a high bit-rate stream into several low bit-rate streams

Robust against frequency selective fading due to multipath propagation.

ISI → 提码宽 → 降低速率 → 频分复用(FDM)

(码元时长小于信道时延) (…大于…) 无ISI低速率 (N路并行同时传输,每路低速率) (不同载波,实现多路并行传输)

$T\uparrow$ $R_B\downarrow$

传统FDM:有保护间隔,耗费了更多的带宽 $\xrightarrow{\text{节省带宽}}$ 正交载波

OFDM基本原理:

设有N个子信道,每个子信道采用的子载波为

$$x_k(t) = B_k \cos(2\pi f_k t + \varphi_k) \quad k=0,1,2\cdots N-1.$$

$$s(t) = \sum_{k=0}^{N-1} x_k(t) = \sum_{k=0}^{N-1} B_k \cos(2\pi f_k t + \varphi_k)$$

$$= \sum_{k=0}^{N-1} B_k e^{j2\pi f_k t + \varphi_k}$$

正交条件: $\int_0^T \cos(2\pi f_k t + \varphi_k)\cos(2\pi f_i t + \varphi_i)\,dt = 0$

推导结果:最小子载频间隔为 $\Delta f_{min} = 1/T_s$ T_s:码元持续时间

$T_s = N \cdot T_B$

频带利用率: $\eta = \frac{N}{N+1}\log_2 M$　当 $N>>1$时　$\eta \approx \log_2 M$

单载波: $\eta = \frac{1}{2}\log_2 M$　大的频道利用率增大两倍

应用&总结: LTE. Wifi;

OFDM的优点.

　　　　时域　　　　　　　　　频域

①抗ISI (frequency selective fading)

② 频谱利用率高

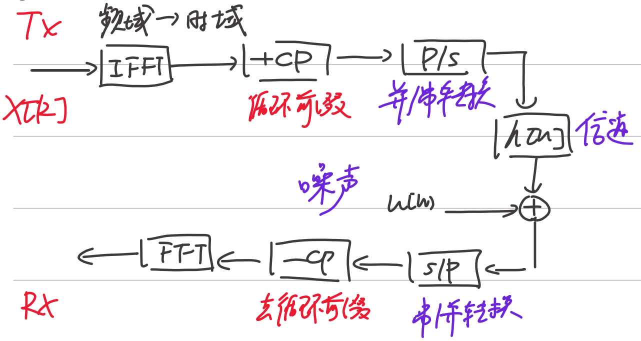
③ 实现灵活.

缺点:

① 对信道产生的频率偏移. 相位噪声很敏感

② 信号峰值功率和平均功率比值较大，这会降低射频功率放大器的效率

Overall structure:



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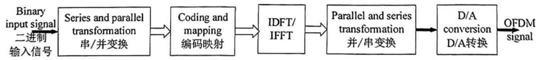
1 OFDM signal can be written as $s(t)=\sum_{k=0}^{N-1} B_k e^{j2\pi f_k t+\phi_k}$, what's the relationship between each subcarrier. The OFDM system transmitter model is shown in Figure 6. In order to realize $s(t)$, which one is the key model.



解: $e^{j2\pi f_k t}$, $e^{j2\pi f_m t}$

In OFDM, the subcarriers are mutually orthogonal over one useful symbol duration T_u.

relationship: $\int_0^{T_u} e^{j2\pi(f_k-f_m)t}\,dt=0,\ k\neq m$

$\Delta f=\frac{1}{T_u}$ $f_k=f_0+k\Delta f$

key module to realize $s(t)$ is IDFT/IFFT block

1 In LTE and 5G system, one of the key technologies is OFDM. What is the meaning of OFDM? What are the advantages of OFDM? Why OFDM can anti-multipath fading?

OFDM: orthogonal Frequency Division Multiplexing.

Advantages: anti-multipath fading, high bandwidth efficiency.

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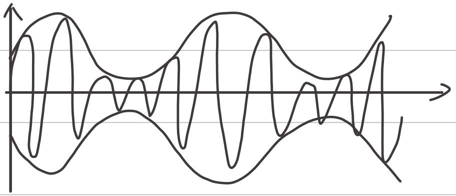
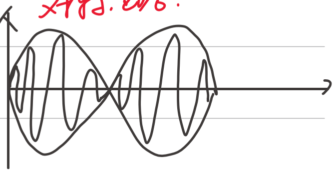
波形:

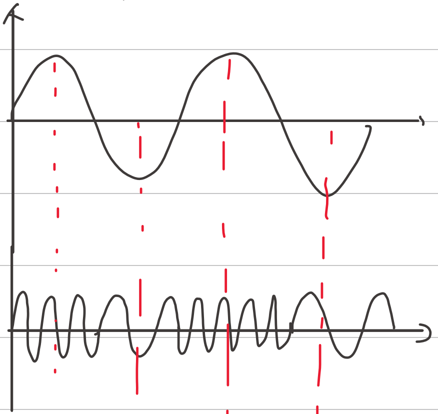
AM 调幅 (调载波幅度) 平移, 对称, 包络



DSB 对称, 包络

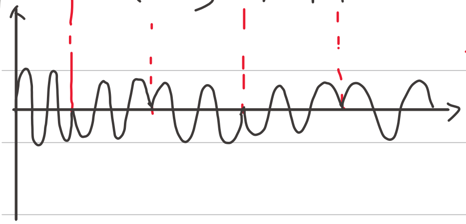


FM 调频 (调制载波的频率)



峰值频率高.
谷值频率低

PM 调相 (调制载波的相位)

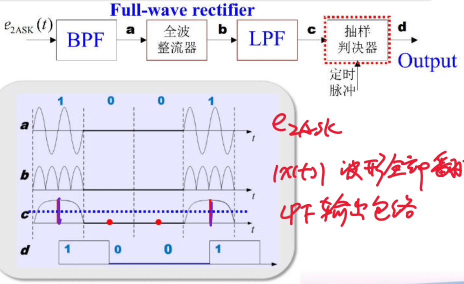


上升
下降 } 相位会翻转

日期: /

§ 7.1.1 Binary Amplitude Shift Keying (2ASK)

◆ Waveforms at Various Points of the Envelope Detection Method



e_{2ASK}

$|x(t)|$ 波形全部翻转到上面去

LPF 输出包络

蓝线：判决门限
红线：抽样时刻 } → 大于 → 1，小于 → 0

Coherence： b: 若发送1： $\frac{A}{2}(1+\cos 2\omega_c t)$ 0 : 0

c. 若发送1： $\frac{A}{2}$ 0 : 0.

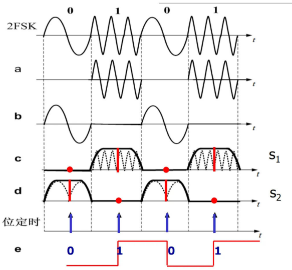
d: 在码元中心进行抽样判决.

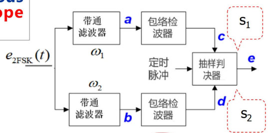
§ 7.1.2 Binary Frequency-Shift Keying (2FSK)

→ 两路2ASK，当 $S_1 > S_{2m}$ 判"1"，$S_1 < S_{2m}$ 判"0"

■ 2FSK Demodulation

◆ Waveforms at various points of the Envelope Detection Method





调制规则：

ω_1 —— "1"

ω_2 —— "0"

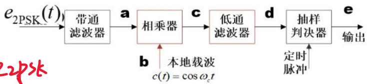
判决规则：

$s_1 > s_2$ 判为 "1"

$s_1 \le s_2$ 判为 "0"

§7.1.3 Binary Phase-Shift Keying (2PSK)

2PSK Demodulation Block Diagram and Waveforms at Each Point：

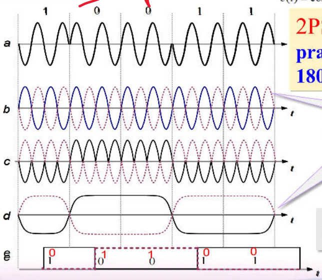


$$\boxed{e_{2PSK}(t)=s(t)\cos\omega_c t}$$

$$\boxed{\cos\omega_c t}$$

$$s(t)\cos^2\omega_c t = \frac{1}{2}s(t)(1+\cancel{\cos 2\omega_c t})$$

→ e2psk



2PSK Existing Problem：In practice, the carrier wave has a 180° phase ambiguity相位模糊.

Carrier Phase Ambiguity

倒π现象
(反相工作)

$\frac{1}{2}s(t)+\frac{1}{2}s(t)\cos 2\omega_c t$

Solution： **DPSK**

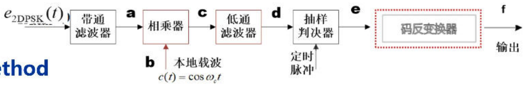
Differential PSK

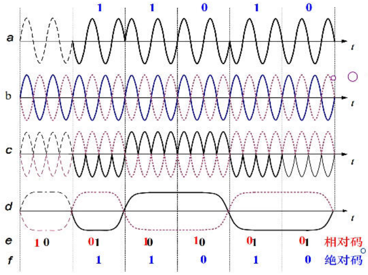
§7.1.4 Binary Differential Phase Shift Keying (2DPSK)

$$\boxed{a_n = b_n \oplus b_{n-1}}$$

(1) 相干解调 + 码反变换法

Coherent Demodulation + Code Inverse Transformation Method





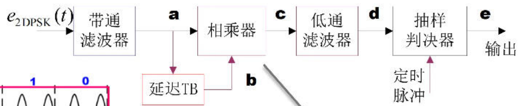
相位模糊
Phase Ambiguity

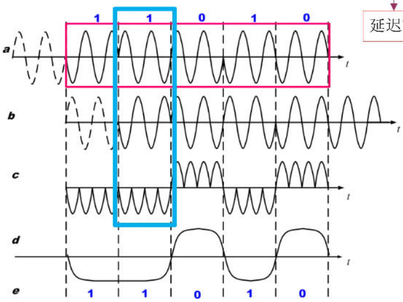
消除影响
Eliminate Impact

§7.1.4 Binary Differential Phase Shift Keying (2DPSK)

(2) 差分相干解调 (相位比较)法

Differential Coherent Demodulation (Phase Comparison)Method





multiplier 起着相位比较的作用

相乘结果 反映了前后码元的相位差

∴不需要差分译码